UNIVERSITY OF SARAJEVO

SCHOOL OF ECONOMICS AND BUSINESS

MASTER THESIS

INVESTING IN COMMODITIES: A VIABLE ADDITION TO AN INVESTOR´S PORTFOLIO?

NEJRA ČUSTOVIĆ

SARAJEVO, MAY 2023

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Sarajevo, Maj 2023

Nejra Čustović

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Potpis studentice

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LIST OF SYMBOLS

ADCC-GARCH - Asymmetric Dynamic Conditional Correlation GARCH Model

CAPM - Capital Asset Pricing Model

CML - Capital Market Line

CBOT - Chicago Board of Trade

COMEX - Commodity Exchange

DCC-GARCH - Dynamic Conditional Correlation GARCH Model

DJCI - Dow Jones Commodity Index

ETF - Exchange Traded Fund

ETN - Exchange Traded Note

HML - Fama-French High Minus Low Returns

SMB - Fama-French Small Minus Big Returns

GMVP - Global Minimum Variance Portfolio

S&P GSCI - Goldman Sachs Commodity Index

GDP - Gross Domestic Product

KCBT - Kansas City Board of Trade

LM - Lagrange Multiplier

LR - Likelihood Ratio

LBMA - London Bullion Market Association

MaxEP - Maximum Return Portfolio

MVP - Minimum Variance Portfolio

MPT - Modern Portfolio Theory

NYMEX - New York Mercantile Exchange

CRB - Reuters/Jefferies Commodity Research Bureau Index

R_f - Risk-Free Rate

RICI - Rogers International Commodity Index

S&P - Standard and Poor's

VaR - Value at Risk

Q-Q Plot - Quantile-Quantile Plot

EXECUTIVE SUMMARY

This thesis aims to provide a detailed analysis on whether or not adding commodities to a portfolio enhances its quality in terms of return and volatility. A special focus on commodities was chosen, due to their hedging capabilities (being that commodities are affected by such factors as weather, economic and political turbulence, supply limitations in production or unplanned increases in demand) and the fact that they are traditionally linked to very low correlations to equity and combinations of equity and bonds, as argued by Jensen and Mercer (2011) and Kayser, Paris and Ross (2011). Substantial empirical research in the field of commodity investment was analysed in order to be able to have a comprehensive overview on what other researchers found out about the diversification benefits of commodities in a portfolio.

This was followed by an empirical analysis, consisting of calculating different portfolios by combining stock, bond and commodity indices over seventeen years, in order to draw a conclusion on whether or not superior risk-return profiles and better portfolio performances are noticeable in portfolios including commodities and how exactly these portfolios are composed. The empirical analysis is conducted with the means of Microsoft Excel and RStudio, for the time period from 02.01.2004 to 01.01.2021, by using weekly prices of four indices that represent three asset classes: the MSCI World Index - Total Return for stock price movements, the FTSE World Government Bond Index - Total Return for bond price movements, the S&P GSCI Commodity Index - Total Return and the Rogers International Commodity Index - Total Return (RICI) Index for commodity price movements. All four total return indices with global character were selected, all in the same currency, in order for them to be comparable, with the timeframe of seventeen years, allowing the commodity investment to be observed and analysed over a longer time period. These four indices are compared in terms of their return, their volatility and their correlation and covariance. Within this empirical analysis, the normality of the data used, i.e. in this case, the indices′ weekly rates of return, was tested with the Kolmogorov-Smirnov test, Shapiro-Wilk test, Q-Q Plot, the skewness and kurtosis analysis and the histogram. To test the research hypothesis of this thesis, three different portfolio optimization calculations were assumed. The first scenario does not contain commodities, i.e. the portfolio is composed exclusively of stocks and bonds. In the second scenario, one commodity index, the S&P GSCI Commodity - Total Return Index, is added to the portfolio, so that the portfolio consists of stocks, bonds and commodities, while in the third scenario, a different commodity index is added to the portfolio consisting of stocks and bonds, the Rogers International Commodity Index - Total Return (RICI) Index, so that the portfolio here also consists of stocks, bonds and commodities. Sharpe ratio and Value at Risk (VaR) are calculated and results of all scenarios analysed and compared in terms of all three scenarios. Additionally, a much shorter, five-year empirical analysis (01.01.2016 to 01.01.2021) was done as well, in order to challenge the findings of the seventeen-year empirical analysis.

The hypothesis of the thesis was not confirmed by the empirical analysis of the paper. While analysing the indices in terms of their return, their volatility, commodities were proven to be a less attractive investment option when compared to stocks and bonds, since the annual rates of return of the two commodity indices were very low, with very high volatility. The correlation analysis concluded that a low correlation of commodity indices was found only in the case of the bond index, the FTSE World Government Bond Index - Total Return. High correlations with the stock index, the MSCI World Index - Total Return, were recorded with both commodity indices, failing to confirm the supposed main advantages of commodities (their very low correlation to equity and combinations of equity and bonds). The five-year empirical analysis (01.01.2016 to 01.01.2021) results in a similar conclusion. While analysing the risk-return profiles of all three scenarios, it is evident that the tangency portfolio (i.e. market portfolio) from the scenario without commodities had the most superior risk-return profile and the highest Sharpe ratio. Thus, even in the second scenario of the seventeen-year empirical analysis, the one with commodities, the maximum Sharpe ratio was found in the portfolio that did not include the investment in the commodity index at all. In the five-year empirical analysis, the portfolio with the highest Sharpe ratio in both *Scenario 2* and *Scenario 3* did not include commodity investments at all, once again supporting the evidence against the hypothesis of the paper. Even though some of the research also claims that adding commodities to a portfolio does enhance its performance, as already mentioned, based on this empirical analysis, it can not be argued that global investments perform better (have a better risk-to-return ratio) if they include commodities and that including this asset class in the portfolio is likely to enhance its performance, due to positive effects of diversification.

In summary, the findings of this thesis are not in line with previous empirical research done in this field and as such offer a good addition to the existing empirical research and the overall argument when it comes to this topic, especially being that it observes portfolio performace over a longer time horizon of seventeen years – through both highs and lows of the commodity market. The time frame of this empirical analysis includes the financial crisis of 2008 and its impact on the markets and ends with the year 2020, the beginning of the COVID-19 pandemic. But how much of an impact exactly the pandemic and the post-pandemic years, complicated further by the changes forced upon the markets by the war in Ukraine, will have on the markets worldwide in the long-run would be a very interesting topic for additional research in this field of study in a few years' time.

Key words: Commodity Investment, Commodity Indices, Diversification Benefits, Risk-Return Profile, Portfolio Optimization

SAŽETAK

Cilj ove teze jeste da pruži detaljnu analizu i odgovor na pitanje da li dodavanje robe u portfolio poboljšava njegov kvalitet kada su u pitanju povrat i volatilnost. Odabran je poseban fokus na robu, radi njenih hedžing svojstava (s obzirom na to da na robu utiču faktori kao što su vrijeme, ekonomske i političke turbulencije, ograničenja ponude u proizvodnji ili neplanirano povećanje potražnje) i radi toga što se roba tradicionalno povezuje s jako niskom korelacijom s tržištem kapitala ili kombinacijom tržišta kapitala i obveznica, kao što tvrde Jensen i Mercer (2011), ali i Kayser, Paris i Ross (2011). Sveobuhvatno empirijsko istraživanje u polju ulaganja u robu je analizirano s ciljem formiranja što jasnijeg pregleda rezultata do kojih su došli drugi istraživači na temu prednosti diverzifikacije robe u portfoliju.

To je potom popraćeno empirijskom analizom, koja se sastoji od kalkulacija različitih portfolija nastalih kombiniranjem indeksa dionica, obveznica i robe kroz vremenski period od sedamnaest godina, kako bi se došlo do zaključka o tome da li su superiorni rizik-povrat profili i bolji performans uopće primjetni u portfolijima koji uključuju robu i kako su ti portfoliji zapravo sastavljeni. Empirijska analiza je sprovedena u Microsoft Excelu i RStudiu, za vremenski period od 02.01.2004 do 01.01.2021, koristeći sedmične cijene sva četiri indeksa koji predstavljaju tri klase imovine: MSCI World Index - Total Return za kretanje cijena na tržištu kapitala, FTSE World Government Bond Index - Total Return za kretanje cijena na tržištu obveznica, S&P GSCI Commodity Index - Total Return i Rogers International Commodity Index - Total Return (RICI) za kretanje cijena na tržištu robe. Odabrana su četiri indeksa potpunog povrata, globalnog karaktera, u istoj valuti (kako bi se mogli međusobno porediti), u vremenskom periodu of sedamnaest godina, što omogućuje promatranje i analizu ulaganja u robu tijekom dužeg vremenskog razdoblja. Kod sva četiri indeksa upoređeni su prinosi, volatilnost, korelacija i kovarijansa. U sklopu ove empirijske analize, normalnost podataka je bila testirana (u ovom slučaju, sedmični prinosi indeksa) i to sa Kolmogorov-Smirnov testom, Shapiro-Wilk testom, Q-Q dijagramom, analizom simetričnosti/asimetričnosti (skewness) i spljoštenosti (kurtosis), kao i histogramom. Da bi se testirala hipoteza ovog magistarskog rada, pretpostavljene su tri različite optimizacije portfolija. Prvi scenarij je scenarij bez robe, dakle portfolio se sastoji isključivo od dionica i obveznica. U drugom scenariju, jedan od indeksa robe, S&P GSCI Commodity - Total Return Index, je dodan portfoliju, tako da se portfolio ovdje sastoji od dionica, obveznica i robe, dok je u trećem scenariju drugi indeks robe, Rogers International Commodity Index - Total Return (RICI) Index, dodan portfoliju, koji se i u ovom slučaju sastoji od dionica, obveznica i robe. Izračunati su Sharpe ratio i Value at Risk (VaR) i sprovedeni poređenje i analiza rezultata sva tri navedena scenarija. Dodatno, značajno kraća, petogodišnja empirijska analiza (01.01.2016 do 01.01.2021) je također urađena, kako bi njen ishod mogli uporediti s onim sedamnaestogodišnje empirijske analize.

Hipoteza nije potvrđena empirijskom analizom ovog rada. Prilikom analize indeksa po pitanju prinosa, volatilnosti, korelacije i kovarijanse, roba je bila manje atraktivna opcija za investiranje u poređenju sa dionicama i obveznicama, radi niskih godišnjih prinosa oba indeksa robe, pri jako visokoj volatilnosti. U korelacijskoj analizi je zaključeno da je niska korelacija indeksa robe nađena u slučaju indeksa obveznica, FTSE World Government Bond Index - Total Return. Visoka korelacija sa indeksom dionica, MSCI World Index - Total Return, je zabilježena kod oba indeksa robe, pri čemu glavna prednost robe (njena vrlo niska korelacija sa tržištem kapitala ili kombinacijom tržišta kapitala i obveznica) nije potvrđena. Petogodišnja empirijska analiza (01.01.2016 to 01.01.2021) rezultira sličnim zaključkom. Prilikom analize profila prinosa-rizika sva tri scenarija, bilo je očigledno da tangentni (tj. tržišni) portfolio iz scenarija bez robe ima najsuperiorniji profil prinosa-rizika i najveći Sharpe ratio. Također, u drugom scenariju sedamnaestogodišnje empirijske analize, onom s robom, najveći Sharpe ratio je imao onaj portfolio koji uopće nije sadržavao robu, što također ne ide u prilog hipotezi ovog rada. U petogodišnjoj empirijskoj analizi, portfolio s najvećim Sharpe ratiom u scenarijima 2 i 3 uopće ne sadržava investicije u robu, što također ne ide u prilog hipotezi ovog rada. Iako neka istraživanja tvrde da dodavanje robe u portfolio poboljšava njegov kvalitet, kao što je već spomenuto, a na temelju ove empirijske analize, nije potvrđena teza da globalne investicije imaju bolji performans (bolji rizik-prinos odnos) ako se u portfolio uključi roba i da će dodavanje ove vrste imovine u portfolio poboljšati njegov performans pozitivnim efektima diverzifikacije.

Dakle, zaključci ovog rada nisu u skladu sa prethodnim empirijskim istraživanjima iz ove oblasti i kao takvi nude dobar dodatak postojećem empirijskom istraživanju i argumentaciji na ovu temu, naročito jer je performans portfolija bio promatran duži niz godina (ukupno sedamnaest), obuhvatajući i uspone i padove tržišta robe. Vremenski horizont ovog empirijskog istraživanja uključuje i finansijsku krizu 2008. godine i njen uticaj na tržišta, i završava sa 2020. godinom i početkom COVID-19 pandemije. Ali koliki će uticaj tačno pandemija i vremenski period nakon nje, dodatno usložnjen promjenama na svjetskim tržištima usljed rata u Ukrajini, imati dugoročno bila bi zanimljiva tema za dodatno istraživanje u budućnosti.

Ključne riječi: ulaganje u robu, indeksi robe, prednosti diverzifikacije, profil rizika i povrata, optimizacija portfolija

1. Introduction

Being that commodities – ranging from precious metals, oils to wood – represent an important source of wealth for many nations, it is no wonder that the topic of commodity investment is becoming more and more attractive to investors worldwide. Investors tipically purchase commodity futures or commodity-linked equity, in order to make use of their expected low correlation with traditional assets and their relation to inflation (Kayser, Paris and Ross, 2011). The main goal of this thesis is to provide its readers with a comprehensive analysis of whether adding commodities to a portfolio enhances the said portfolio's quality – in terms of return and volatility. Sub-chapters 1.1. and 1.2. focus on the overview of the research problem and subject, as well as the literature review. The research hypothesis and methodology, along with the purpose and objectives of the thesis and the structure of the thesis, close out this introductory, first chapter.

1.1. Subject and research problems

Despite the fact that investing in physical commodities is becoming increasingly less attractive, there was a significant increase in interest in commodity investment over the past years due to a spike in interest in exchange-trades commodity products (Jensen and Mercer, 2011). The suspected reason for this ever-increasing interest in commodities lies in commodities being strongly influenced by factors such as weather, economic or political turbulences, increase in demand in emerging markets and the supply limitation when it comes to commodity production, as well as other geopolitical circumstances – unlike traditional asset groups. However, when discussing the rising interest in commodity investment, there are two important factors that need to be outlined as the driving force behind it: (1) the increased variety of investment instruments when investing into commodities and (2) the poor performance and increased volatility of equities and real estate after the technological bubble in the year 2000 and the economic crises of 2008-2009 (Jensen and Mercer, 2011). With a relatively poor performance of traditional asset classes on one hand and the increasing performance of commodity market indices – for instance, that of the Rogers International Commodity Index (RICI), with its astonishing increase in total return of 324.00% in the time period August 1998 until April 2022 – on the other, it is hardly surprising that investing in commodities is becoming increasingly interesting for investors worldwide (Beeland Interests, 2022).

Therefore, one of the main advantages of commodities is the fact that they are traditionally linked to very low correlations to equity and combinations of equity and bonds (Kayser, Paris and Ross, 2011). As argued by Carmona (2015), a part of the reason for this lack of correlation is the inflation – which is why investing in commodity futures is regarded as an inflation hedge, since a rise in demand for goods and services (usually in periods of increasing inflation) inevitably also results in a rise in demand for commodities, being that those same commodities are used in the actual production of the abovementioned consumer goods and services in the first place, as opposed to financial instruments such as stocks and bonds, that have a decreasing performance in times of increased inflation (Carmona, 2015).

In the time period 2002-2008, there was a significant increase in demand in commodity markets, followed by an increase in prices. In case of certain commodities, this said increase in prices was halted with the beginning of the recession in the USA and Europe by the end of 2007, while prices of other commodities have continued to rise until mid-2008, when the downturn in the economy led to a decrease of both demand for commodities and commodity prices (Östensson, 2012). After the financial crisis of 2008, from the second quarter of the year 2009 onwards, the prices across commodity markets began to rise significantly, which is not consistent with past economic declines, after which prices dropped or increased marginally at best (Helbling, 2009).

1.2. Literature review

A detailed analysis of whether adding commodities to a portfolio enhances the said portfolio's quality – in terms of risk and return – is the main goal of this thesis. In order to evaluate the benefits of adding commodities to a portfolio, along with all factors that need to be considered when looking to invest in commodities, a number of articles and research papers was analysed, with each of said papers having a slightly different focus and hence providing an important contribution to an overall more complete and multidimensional view of the topic. The rest of Sub-chapter 1.2 consists of short summaries and most important points of articles and research papers used for the creation of this thesis.

According to the research of Heidorn and Demidova-Menzel (2011), the limitation in gold sales by central banks lead to a decrease in supply and increased gold prices. And even though gold prices started to increase, due to a low correlation to equity, gold was still largely considered an attractive investment opportunity from the year 2000 onwards (Heidorn and Demidova-Menzel, 2011).

However, different risk reduction capabilities have been observed with different precious metals over different time periods. In the medium and short term, investing in gold leads to a strong downside risk decrease. On the other hand, silver and platinum show strong risk reduction characteristics only in the short term. In the long term, silver and platinum may even lead to a higher downside risk for higher allocations, as implied by Bredin, Conlon and Potì (2017). A decreased Sharpe ratio and a modified Sharpe ratio of a portfolio including precious metals, compared to an equity-only portfolio, in the period 1980-2014, is observed in their research. As per their research, investors are willing to give up on returns in order to decrease the probability of high tail-losses, contradicting earlier research in the field. According to Bredin, Conlon and Potì (2017), investing into ETFs and futures of precious metals is considered a good diversification option, next to investing in physical metals.

On the other hand, according to the research of Belousova and Dorfleitner (2012), no overall conclusion can be made for all commodities analysed when it comes to the question of which commodity to choose – however, differences between them are observed. Even though the most valuable addition to a portfolio are commodities from the energy and precious metal sectors – since they are beneficial for the portfolio in both bear and bull markets in terms of both risk and return, making them a good choice for both conservative and aggressive investors – risk reduction capabilities are observed in other sectors as well. Risk reduction in a portfolio is observed upon adding agricultural, livestock and industrial metal commodities to a portfolio, which is particulary attractive to risk-adverse investors, being that the allocation is more on the conservative part of the efficient frontier (near the GMVP). Weak diversification capabilities are noted in softs, grains and livestock sectors, as observed in bear markets, whereas industrial metals have almost no diversification capabilities in bull markets (Belousova and Dorfleitner, 2012).

Portfolio diversification capabilities do not just vary depending on the type of commodity, but also on the time period (for instance, there are noticeable differences when comparing tranquil and turbulant periods, periods of financial crises etc.), as per Öztek and Öcal (2017). According to their research, the increase in correlation between the agricultural commodity sub-index and the stock market index is mostly due to financial crises, complemented by relatively high market volatilities. Additionally, the agricultural commodity market is regarded as a better opportunity for portfolio diversification in the midst of more tranquil periods. Increasing trends between the precious metal sub-index and the stock market index are noted, but their correlation levels are highly affected by market volatilities throughout financial crises. In summary, the research suggests that high gains arise from portfolio diversification between commodity and stock markets as opposed to investing solely in the stock market. Thus, according to the research in question, the portfolio performs better during more tranquil periods than during more volatile ones. Also, the research indicates that markets are not integrated enough to have a lasting trend in the correlation and that they are rather dependent on market volatility – and that the optimal weights of assets in the portfolio should be tailored in accordance with the market regimes (Öztek and Öcal, 2017).

When it comes to investing in commodities, the main factors influencing the returns of the commodity futures market should be considered as well. According to Main, Irwin, Sanders and Smith (2018), upon analysing whether the risk premium of long-term commodity investors was affected by the financialization of commodity futures markets or not, it is concluded that the returns in commodity futures markets are mainly driven by random individual supply and demand fluctuations. The conclusion is supported by the finding that the average level of unconditional risk premiums was mainly not influenced by the financialization that flooded the commodity futures market in the mid-2000s (Main, Irwin, Sanders and Smith, 2018).

Aside from the commodity type, the investment period and influences on the returns of the commodity market also need to be considered when investing into commodities. The interaction of other asset classes with the commodity asset class should be taken into account as well. The argumentation for an interaction of bond, equity and commodity cycles is provided in the research of Narayan, Thuraisamy and Wagner (2017), implying that macroeconomic uncertainty influences equity and then the bond market and consequently the measure of uncertainty as well. Therefore, there is a lagged cross-market pricing transmission from gold to bonds and – consequently – to oil and inflation.

When trying to answer the question of whether or not adding commodities to a portfolio contributes to the quality of the said portfolio in terms of risk and return, the main focus lies on the topic of diversification benefits of commodities. The article authored by Cotter, Eyiah-Donkor and Potì (2017) provides an in-depth analysis of this topic. However, it should be noted that in-sample and out-of-sample analyses have resulted in different findings. This may be due to the fact that in-sample analyses use flawless expected return forecasts, which might exaggerate the diversification capabilities of portfolios and in turn lead to false diversification results. The in-sample analysis showed that adding commodities, currencies or both to a traditional portfolio does indeed lead to diversification benefits – however, primarily close to the global minimum variance share of the frontier. This, however, does not apply to the period of the commodity 'boom', as that was the period during which commodities did not have diversification benefits. Higher Sharpe ratios were observed in the portfolio including commodities, currencies or both, when return predictability was accounted for. However, the out-of-sample analysis showed that there are no diversification benefits resulting from including currencies or commodities.

On the other hand, in the article authored by Cheung and Miu (2010), it is concluded that diversification benefits of investing in commodities are either regime-dependent or (investment) period-dependent. As per said research, commodities provide statistically significant diversification benefits in the long run. Additionally, it is noticeable that changes in the behavior of commodity futures are regime-dependent (low return commodity futures environment is linked to low volatility and vice versa). In bearish stock markets, diversification benefits of commodities are very low, which leads to the conclusion that the real advantage commodities provide in the not-so-frequent outbreaks of the commodity market is via virtue of increasing the portfolio performance. Lastly, it is observed that diversification benefits are not generally applicable and that investing into commodity futures is a better fit for risk-averse investors (Cheung and Miu, 2010).

There is no universal conclusion to the discussion regarding the advantages and disadvantages of commodity investment and its hedge benefits, especially as they vary depending on the type of commodity and the changing dynamics between equity and bond markets, the oil and gold markets and the volatility index – and their periodical dependencies. Abid, Dhaoui, Goutte and Guesmi’s (2019) research implies that the hedging of investors’ risk exposure can also be done by investing into gold, oil and bonds, due to their 'safe haven' properties. Furthermore, strong diversification capabilities of commodities are also highlighted. The best hedge for the U.S. equity market is provided by gold, with this precious metal proving to be a better hedge than bonds or oil in the long run. It is mainly those assets (oil, gold and bonds) that have shown cycles of positive reliance with equities, underscoring the hedging benefits of such assets even more. Additionally, it is concluded that the volatility index and equity markets have opposite dynamics, implying massive diversification benefits. Nevertheless, it is additionally outlined by Abid, Dhaoui, Goutte and Guesmi (2019) that a good selection of company size and portfolio selection based on its profitability might result in the best hedging benefits.

On the other hand, Fethke and Prokopczuk (2018) argue that diversification benefits of commodities differ with different generations of commodity indices as well, since their research includes data of commodity indices of the first, second and third generation. It is concluded in their research that second and third generation indices are better linked to higher diversification benefits than those belonging to the first generation - however, the wide reaching sample indicates variation within the group of those indices.

All of the abovementioned research provides a very good overview of not just all factors that need to be considered when investing in commodities (such as commodity type or investment period), but also a solid research on how to evaluate diversification benefits of commodities, especially when compared to other, more traditional asset classes, like stocks or bonds. This overview, along with the emipirical analysis in Chapter 4, should provide a decent basis for deciding on whether adding commodities to a portfolio enhances the said portfolio's quality in terms of risk and return – or not.

The timeframe of the said empirical analysis includes the years from 02.01.2004 to 01.01.2021 (i.e. the seventeen-year empirical analysis). According to the researchers and their articles and research papers mentioned previously in this Sub-chapter, the year 2004 is a fairly significant one, being that it is when European central banks agreed to limit the gold sales going forward, leading to lower supply and higher prices of gold (Heidorn and Demidova-Menzel, 2011). In order to include these effects, as well as the effects of the financialization of the commodity futures markets, which also occurred in 2004 (Main, Irwin, Sanders and Smith, 2018), the year 2004 was decided upon as the first year in the timeframe of the empirical analysis in this thesis. The end year is the year 2020, so that the most recent macroeconomic turbulences, set in motion primarily by the outbreak of the COVID-19 pandemic, are taken into account as well – at least those pertaining to this first COVID-19-stricken year. A much shorter, five-year empirical analysis (0**1**.01.2016 to 01.01.2021) was done as well, in order to challenge the findings of the abovementioned seventeen-year empirical analysis.

1.3. Research hypothesis

The research hypothesis of this thesis reads as follows:

Hypothesis: <u>Global investments perform better (have a better risk-to-return ratio) with commodities, than without them.</u>

Therefore, including commodities in the portfolio enhances the portfolio performance, due to positive effects of diversification, which is why global investors' portfolio should include commodities in their respective portfolios.

The said hypothesis is tested empirically in the fifth chapter of this thesis.

1.4. Purpose and objectives of research

The purpose of this thesis is to provide its readers with a detailed analysis of investing in commodity markets and whether or not investing in commodities is a good alternative for today's investors. In other words, this thesis aims to provide a detailed analysis of whether or not adding commodities to a portfolio enhances the said portfolio's quality – in terms of return and volatility. This thesis aims to provide potential investors with an overview of the empirical research and analysis, in addition to a theoretical framework that covers the essentials of trading in commodity markets as well as the Modern Portfolio Theory (MPT). With its empirical analysis, this thesis aims to offer a significant contribution to the field of alternative (in this case – commodity) investments and portfolio optimization.

The research objectives of this thesis are…:

a) …to provide a theoretical background concerning commodity markets and investing;
b) …to provide a theoretical background concerning the Modern Portfolio Theory (MPT) and its advantages and disadvatages;
c) …to provide a good and concise overview of the empirical research in this field so far;
d) …to provide a reliable empirical analysis over a longer period of time (seventeen years);
e) …to provide the information on whether or not commodities contain less risk than other forms of assets;
f) …to provide the information on whether or not commodities gain higher returns than other forms of assets;
g) …to provide the information on how commodities correlate with other forms of assets;
h) …to provide the information on whether the rates of return of the indices analysed in the empirical analysis are normally distributed;
i) …to provide a conclusion on whether or not it is ultimately beneficial to include commodities in the portfolio and, if so, how such a portfolio would/should be structured;
j) …to provide a conclusion on whether or not there are differences in portfolio optimization depending on the selection of commodity indices.

1.5. Research methodology and limitations

This thesis consists of two main sections: the empirical research (see Chapter 4) and the empirical analysis (see Chapter 5). Two different research methodologies were used in these two sections. Analytical research was used in the first section – and consists of a comprehensive analysis of ten relevant empirical research papers, with the goal being to gain understanding of what prominent researchers and scholars have to say and contribute in this regard. The second section consists of the empirical analysis, in which quantitative research was conducted, so as to test the research hypothesis presented in Sub-chapter 1.3. Due to the difference in research methodology used and limitations between these two sections, they are outlined separately below.

Empirical research

The selection of the empirical research papers used in this paper was done by the author of this thesis, based on the research topics and timeframes analysed. These empirical research papers used, explained in detail in Chapter 4, offer a good summary of factors to consider when investing in commodities (i.e. commodity type or investment period), as well as insights into the topic of diversification benefits of commodities, especially in comparison to other traditional asset classes such as stocks or bonds. The main and perhaps most obvious limitation of this approach is the number of empirical studies used and that the selection of these empirical research papers was done by the author of this thesis. Even though only the most relevant research papers were selected for this thesis, with each empirical research paper focusing on a specific topic under the umbrella of commodity investing and hence approaching it from a slightly different angle and, in doing so, contributing to answering the research hypothesis, the selection of the empirical research papers was nevertheless solely based on the judgement of this thesis' author.

Empirical analysis

This empirical analysis is a quantitative research, conducted with the means of Microsoft Excel and RStudio, for the time period of seventeen years, from 02.01.2004 to 01.01.2021, by using weekly prices of four indices that represent three asset classes (stocks, bonds and commodities). Additionally, a much shorter, five-year empirical analysis (01.01.2016 to 01.01.2021) was done as well, in order to challenge the findings of the seventeen-year empirical analysis.

Four total return indices with global character were selected – all in the same currency, in order for them to be comparable. The data sample used in the seventeen-year empirical research covers the indices' weekly price movements in the period 02.01.2004 until 01.01.2021 (seventeen years), which allows for commodity investment to be observed and analysed over a longer time period. On the other hand, the data sample used in the five-year empirical research covers the indices' weekly price movements in the period 01.01.2016 to 01.01.2021 (five years).

Following indices were selected to represent the abovementioned three asset classes:

a) MSCI World Index - Total Return for stock price movements;
b) FTSE World Government Bond Index - Total Return for bond price movements;
c) S&P GSCI Commodity Index - Total Return for commodity price movements;
d) Rogers International Commodity Index – Total Return (RICI) Index for commodity price movements.

The seventeen-year empirical analysis can be divided in three main parts:

1. Risk and return analysis

The abovementioned four indices are compared in the time period 02.01.2004 until 01.01.2021 (seventeen years in total) – in terms of their return and their volatility. Additionally, the correlation and covariance between these three asset classes were calculated.

2. Data normality tests

The normality of the data used (in this case, the indices′ weekly rates of return) are tested with the two most commonly used methods: the Kolmogorov-Smirnov test and the Shapiro-Wilk test. Additionally, this empirical analysis also includes Q-Q Plots, skewness and kurtosis analyses and histograms.

3. Portfolio optimization

To test the research hypothesis of this thesis, different portfolio optimization calculations (i.e. three different scenarios) were assumed and their results analysed:

Scenario 1

In the first scenario, the portfolio is composed exclusively of stocks and bonds.

Scenario 2

In the second scenario, the S&P GSCI Commodity - Total Return Index is added to the portfolio, so that the portfolio consists of stocks, bonds and commodities.

Scenario 3

In the third scenario, the Rogers International Commodity Index - Total Return (RICI) Index is added to the portfolio, so that the portfolio consists of stocks, bonds and commodities.

For all scenarios, Sharpe ratios and Value at Risk (VaR) are calculated and results of all scenarios analysed and compared in terms of all abovementioned indicators.

The much shorter, five-year empirical analysis (01.01.2016 to 01.01.2021) includes a risk-return profile analysis, correlation and covariance calculations and portfolio optimization calculations for the same three scenarios as in the seventeen-year empirical analysis, focusing however just on the risk-return profiles of each randomly assumed portfolio and their Sharpe ratio, in order to challenge the findings of the seventeen-year empirical analysis.

The main limitations of the empirical analysis (see Chapter 5) conducted are as follows:

- Four indices are chosen to represent the price movements of three asset groups

  The biggest limitation of this thesis is that four indices are chosen to represent price movements of three different asset groups (stocks, bonds and commodities). All indices used have a global character and are in the same currency. However, the research could possibly be improved by using more indices per asset class (not just in case of commodities), in order to get more reliable data or even add additional asset classes to the research. Opting for four indices instead was done in order to avoid unnecessarily complex data sets, since the calculations are done mainly in Microsoft Excel and not in a statistical calculation program.

- Weekly price movements are observed

  One further limitation of this empirical research is that weekly price movements, instead of daily price movements, are used. If daily prices were used, the research would likely result in more reliable data. However, the sheer amount of data would be too complex for processing, which is why weekly prices are used instead.

- A seventeen-year timeframe is used

  A seventeen-year timeframe is used, from the beginning of 2004 until the end of 2020. In order to improve the quality of the empirical analysis, even longer timeframes could be selected, in order to obtain more reliable and accurate data. The reasons for choosing this particular seventeen-year timeframe are outlined in Sub-chapter 5.1. In order to challenge the findings of the seventeen-year empirical analysis, a shorter, five-year empirical analysis (01.01.2016 to 01.01.2021) was conducted as well.

- Limited number of indicators is calculated

  Since the calculations are done partly in Microsoft Excel and partly in RStudio instead of an entirely statistical calculation program, only a limited number of indicators (such as risk, return, correlation, covariance, Sharpe ratio, Value at Risk etc.) is calculated. In order to expand on this, additional measures, tests and statistics may be used.

1.6. Thesis structure

This thesis consists of six chapters.

The first chapter, the introduction, aims to provide an overview of the subject of the thesis and the literature used. Additionally, the focus is also on the research hypothesis and methodology, as well as its limitations.

The second chapter focuses on the essentials of trading on the commodity market. The definition and classification of commodities, trading of commodities, as well as the most important commodity indices are outlined and evaluated in detail.

The third chapter aims to provide a detailed explanation of the Modern Portfolio Theory (MPT) by Markowitz – along with its most prominent advantages and disadvantages.

The fourth chapter provides an overview of the empirical research available in the field of commodity investment. Additionally, the reasons for and against investing in commodities are discussed as well.

The fifth chapter revolves around the empirical analysis, in which different indices are used so as to represent different asset groups (stocks, bonds and commodities). A risk and return analysis as well as a data normality analysis of the abovementioned different asset groups are presented. In addition, a portfolio optimization calculation by Markowitz is calculated so as to compare different types of portfolios: one with and one without commodities. Furthermore, two different portfolios including commodities were calculated, each including a different commodity index, so that the difference in the portfolio optimization depending on the selection of the commodity index can be properly evaluated as well. A longer, empirical analysis was conducted over a time horizon of seventeen years (02.01.2004 to 01.01.2021), along with a much shorter, five-year empirical analysis (01.01.2016 to 01.01.2021), in order to challenge the findings of the original, seventeen-year empirical analysis.

The sixth chapter is the thesis' conclusion. From the information and data shared over the course of the thesis up until this point, including data from the empirical part of the thesis, final thoughts on whether or not to add commodities in an investor's portfolio are shared in this part. Additionally, new trends and latest insights in this field are outlined as well.

2. Essentials of trading on the commodity market

Raw and primary products are traded on commodity markets instead of manufactured products (Teall, 2018). Due to the obvious disadvantage of having to hold actual physical commodities and the cost of it, investing in commodities as an asset class became more popular only fairly recently. The rising interest in commodities is owed in part to their hedging capabilities, being that commodities are affected by such factors as weather, economic or political turbulence, supply limitations in production or unplanned increases in demand – as opposed to other asset classes that are not as influenced by those same factors. Commodity investment becoming more popular among investors is also driven by the increasingly attractive offer of commodity instruments to choose from – as well as (indirectly) by the high risk and low return of real estate and equity after the crises of the years 2000 and 2008 (Jensen and Mercer, 2011).

In order to help provide a better understanding of the said increasing popularity of commodities, this chapter provides an overview of the most importantl characteristics of commodities and their classification. Additionally, various investment opportunities – when it comes to commodities and the relevant commodity indices – are outlined as well, in order to provide the reader with an overview of the essentials on trading on commodity markets.

2.1. Essentials of the commodity sector

Per definition, commodities are homogeneous products traded among consumers and producers and typically include agricultural goods, fuels, minerals, metals and forestry products (World Trade Organization, 2010). However, for trades on developed commodity markets, straightforward and standardized commodities are needed, i.e. it is irrelevant who the owner or seller of the commodity is as that has no influence on the commodity itself (Teall, 2018).

Due to their specific characteristics, investing on the global commodity market is significantly different than investing in other assets – with some of the reasons being, according to Fabozzi (2008):

- ***Real assets***

  Commodities are considered to be *real assets*, meaning that they are consumption goods first and foremost, and not investment goods – and that they provide value by being used in industrial manufacturing or in consumption.

- ***Limited supply***

  One of the specific characteristics of commodities is their limited availability, meaning that there is a cap in the amount of commodities that can be produced (i.e. annually harvested amount of wheat). Additionally, some commodities cannot be produced/harvested during the entire year and are therefore seasonal (i.e. agricultural commodities are seasonal, while metals can be mined virtually all year long).

- ***Storability and availability***

Storability and availability are very important characteristics, as they have a massive influence on the price of a given commodity. Commodities can be classified into storable (with a high degree of storability, non-perishability and low storage costs) and non-storable commodities. An example of commodities with a high degree of storability would be industrial metals, in stark contrast to – for instance – livestock, which is considered to have a limited level of storability.

- ***Heterogeneity***

Being that every type of commodity has its own specific characteristics and their quality is therefore not standardized, commodities are considered to be heterogeneous (Fabozzi, 2008). The risk and return profiles of the commodity sector can differ quite significantly, and may possibly not even move in the same way, due to the said heterogeneity (Anson, Fabozzi and Jones, 2011).

2.2. Classification of commodities

Commodities are usually classified into hard and soft commodities, with hard commodities being products from the energy, precious metal, and industrial metal sectors, while soft commodities are perishable commodities from the agricultural sector as well as weather-dependent commodities like grains, soybeans, or livestock (Anson, Fabozzi and Jones, 2011). According to Eller and Sagerer (2011), hard commodities include energy (fossil, nuclear and alternative energy) and metals (precious, base and ferrous metals), whereas soft commodities include food and consumer products (wheat, oilseeds, semi-luxury goods), industrial agro-raw materials (cotton, wool, timber, rubber) and animal agro-raw materials (feeder cattle, live cattle, lean hogs). There is a further classification of commodities – in primary and secondary commodities – depending on whether the commodities are derived from nature directly (for example, crude oil) or manufactured from primary commodities (for example, gasoline produced from oil), according to Harasheh (2021).

2.2.1. Hard commodities

Hard commodities (like oil, gold, natural gas or rubber) are usually extracted commodities (Teall, 2018). Due to their crucial role in manufacturing and production as well as their higher manageability in comparison to soft commodities, the markets of hard commodities are extremely liquid and are therefore the traders' first choice when it comes to trading commodities (Yadav, 2018). As per Eller and Sagerer (2011), they consist of energy and metals – with energy being divided into fossil and alternative energy, and metals into precious, base and ferrous metals. According to Yadav (2018), a handful of agricultural products, i.e. soft commodities (like cotton) can be viewed as hard commodities, due to their lack of perishability. As argued by Eller and Sagerer (2011), since there is a limited quantity of fossil energy on Earth, rendering this resource non-renewable, its market value is not solely shaped by the market's usual demand/supply equation, but by its global volume as well. Thus, rising prices of hard commodities are likely in the future, due to the increasing demand and limited supply.

Energy

Gasoline, propane, gas, crude oil, heating oil, coal, ethanol, natural gas and electricity are considered energy commodities (Yadav, 2018). According to Eller and Sagerer (2011), the alternative energy sector is becoming increasingly more important, due to the scarcity of fossil resources worldwide, while crude oil still accounts for the largest share of all fossil energy (up to 45% in the year 2005), making it the most important energy source currently available.

However, due to the limited nature of fossil resources, there is a greater emphasis on alternative energy. The price of crude oil is expected to increase in the long run, as a result of to the mentioned scarcity of fossil resources. Since crude oil cannot be used in its natural form, it needs to be processed in refineries prior to being used (Eller and Sagerer, 2011).

Energy prices demonstrate high volatility over the years, due to diverse political, financial and economical influences (like the financial crisis in 2008/2009 and its negative influence on the economy, resulting in a decrease of demand; the decline of commodity prices in 2015/2016 due to China's declining economic growth trend and its important role in the demand side of energy commodities or the decline of demand at the beginning of the COVID-19 pandemic, resulting in a price decline of energy commodities), as argued by Harasheh (2021).

Eller and Sagerer (2011) argue that the price of crude oil in the long run depends not solely on supply and demand but also on the available volume of this resource as well. The OPEC (Organization of Petroleum Exporting Countries) controls the supply of crude oil, by determining the delivery and price levels of crude oil of member countries. On the other hand, the demand side is heavily impacted by the demand of the ten biggest countries/consumers, that use up to 60% of the total amount of crude oil worldwide. Even though the roles of China and India have increased considerably over the last years, it is the USA alone that use an astonishing 25% themselves, making them the single biggest crude oil consumer in the world. The daily crude oil demand has increased by more than 20% in the time period 1980-2006, amounting to 85 million barrels per day today (Eller and Sagerer, 2011). On the supply side, according to Huang (2019), significant exporters of energy like natural gas and crude oil are the countries of the former Soviet Union. It should be noted that the Middle East's economy relies heavily on crude oil exports, for example 90% of all exports in Saudi Arabia and 84% of all exports in Iran come from oil (Huang, 2019).

When it comes to pricing/trading of crude oil, speculation has a significant impact in the short run. According to Eller and Sagerer (2011), an insignificant volume of crude oil is traded on the spot market (Rotterdam and New York), whereas a more significant volume is traded on over-the-counter markets (New York, London, Singapore, and Tokyo), with the trading volume exceeding the actual daily production volume of crude oil. Seasonal impact on the trading of crude oil should be considered as well, with the months of March, April and July to September being especially preferred for the holding of a tactical long position in oil (Eller and Sagerer, 2011). According to Huang (2019), the most significant exporters of crude oil are Russia, Mexico, the Middle East, Nigeria, Venezuela and Scandinavia, while USA, South Korea, Europe, China and Japan are the most significant importers of crude oil worldwide.

Eller and Sagerer (2011) note that natural gas develops in a similar direction as crude oil, with both resources characterized by their limited supply and increasing demand. According to recent research, the supply of crude oil should suffice for another 43 years, while in case of natural gas supply it is 65 years. Both crude oil and natural gas reached all-time low price levels in the years 1998-1999, and their price has been on the rise ever since. A strong positive correlation between those two hard commodities is apparent (Eller and Sagerer, 2011). According to Huang (2019), the most significant exporters of natural gas are Russia, Nigeria, Indonesia and Saudi Arabia, while Europe, Turkey, Japan and South Korea are the most significant importers worldwide.

Due to natural resources being limited, and their price increasing, alternative energy sources keep gaining importance. Use of renewable energy is expected to become an increasingly popular trend in upcoming years (Eller and Sagerer, 2011).

Metals

The driving force of development, whether economic or human, is metals (Harasheh, 2021). Metals consist of precious metals (gold, silver, platinum etc.), base metals (aluminium, copper, nickel etc.) and ferrous metal (iron, steel), as outlined by Eller and Sagerer (2011).

According to Eller and Sagerer (2011), gold is traded in form of futures contracts at the Commodity Exchange (COMEX) in New York, while it is traded as actual physical gold at the London Bullion Market Association. The most significant supplier of gold is South Africa with 15% and the USA with 11% - with their gold coming mainly from mining companies and central banks. The biggest demand for gold still comes from the jewelry industry (more than 80%), while investments in gold make up for a rather insignificant portion of overall demand (about 2%). Ever since 1999, when the price of gold reached its all-time low, the price level of this precious metal has been on a steady rise. The months from May to September and December to February are known to be the most typical for the holding of a long position in gold (Eller and Sagerer, 2011).

Recently, however, in the beginning of 2020, the COVID-19 pandemic influenced a decrease in demand for metals, resulting in a decrease of main metals' prices, like aluminum, nickel, copper, lead, iron, zinc etc. (Harasheh, 2021).

As argued by Eller and Sagerer (2011), the economic development strongly correlates with the demand for base metals, or the so-called industrial metals, since they are used in the building industry, the most significant representative of this group being aluminium, characterized by high costs of production and high correlation with oil prices.

One of the most significant exporters of aluminium besides Russia, is Australia (Huang, 2019). However, according to Eller and Sagerer (2011), the most significant supplier of aluminium worldwide is China (20%), followed by Russia (13%) and the USA (10%). The USA and Europe are the most notable contributors when it comes to demand, especially in the automotive industry. Same as with gold, aluminium prices have had an increasing trend from the year 1999 onwards (Eller and Sagerer, 2011).

2.2.2. Soft commodities

Agricultural products like sugar, wheat, coffee, livestock, cocoa are considered soft commodities (Teall, 2018). According to Eller and Sagerer (2011), soft commodities consist of food and consumer products, industrial and animal agro-raw materials. They are a renewable resource, meaning that their price is determined by supply and demand on the market, as opposed to the above mentioned hard (and non-renewable) commodities. While hard commodities have experienced an increase in their price levels over the last years, soft commodities are at their lowest level price-wise, despite the ever-increasing demand. However, when talking about the increasing demand for soft commodities (owed largely to the increasing consumption of the population), it needs to be said that it is expected to lead to an increase in price levels of soft commodities in the long run. External effects such as global industrialisation, changing nutritional habits of the population and increased consumption of the population (due to its growing size) are all expected to drive this price increase of soft commodities in the future (Eller and Sagerer, 2011).

Food and Consumer Products

Wheat is the most essential commodity of all food and consumer products, since it is being used in the production of flour, beer and whiskey – closely followed by corn, soybeans, coffee beans, cocoa and sugar (Eller and Sagerer, 2011).

Even though sugar can be found in various parts of the world, Brazil, Mauritius, Australia, Thailand and Cuba are the most significant sugar exporters worldwide, with Brazil having a market share of 30%, making it the country with the biggest sugar export in the world

(Huang, 2019). Russia is currently the most significant sugar importer in the world and Asia is increasingly gaining in importance as an importer (Huang, 2019).

The European Union (18%), China (16%) and India (13%) are the world's most significant suppliers of wheat, while the highest demand for wheat comes from China (17%) and the European Union (16%), as per Eller and Sagerer (2011).

As mentioned earlier, wheat is closely followed by corn importance-wise, and can be found almost everywhere. It is usually used in form of animal food as well as in food production (alcohol, sweets etc). The world's annual corn production stems mostly from the USA (38%) and China (20%), followed by Brazil and the European Union (Eller and Sagerer, 2011), while the highest demand for corn is also noted in the USA (32%) and China (20%), meaning that a mere fraction of this commodity actually ends up being exported to international markets (Eller and Sagerer, 2011).

Industrial Agro-Raw Materials

Even though cotton, as the most important representative of industrial agro-raw materials, is grown in more than 70 countries, China (25%) and the USA (20%) are the most significant producers of cotton in the world, which is then predominantly used in the textile industry, while the highest demand for cotton comes from China and Indonesia, making these two countries the most significant importers of cotton worldwide (Eller and Sagerer, 2011).

Animal Agro-Raw Materials

Overall, when it comes to meat, the most significant exporters are countries like the USA, Brazil and Argentina, while the most significant importers are Europe, North-East Asia, Japan, Middle East and Russia (Huang, 2019).

When it comes to cattle beef, the most significant supplier is the USA (25% of the global production), followed by Brazil (16%), the European Union (15%), and China (12%), while the USA, Brazil, the European Union and China also have the highest demand for cattle beef (Eller and Sagerer, 2011). Pigs ready for slaughter, or the so-called lean hogs, are another significant animal agro-raw material. China is the most important supplier (with 50% of overall supply), as well as the biggest consumer, which is why this country exports very little of this resource, while the European Union, on the other hand, with its production reaching the 20% mark, is the world's biggest exporter of pork (Eller and Sagerer, 2011).

2.3. Commodity investment opportunities

The limitations of investing in physical commodities result in an increased popularity of investing in the commodity asset class (Jensen and Mercer, 2011). According to Chambers, Black and Lacey (2018), a significant share of invested assets of institutions is assigned to commodities, lending even more credibility to the argument of increased popularity of this asset class among investors. For instance, the CAIA Alternative Index assigns 10% of its weight to commodities from the realm of institutional alternatives (Chambers, Black and Lacey, 2018). However, there are different ways to invest in commodities, aside from the most obvious one (directly buying physical commodities), for both private and institutional investors (Jensen and Mercer, 2011). Proof of rice futures trades is found in China 6000 BCE, meaning that futures contracts were used centuries before the stock markets emerged (Teall, 2018).

This chapter provides an overview of some of the predominant ways and forms of investing into commodities – including (Carmona, 2015):

Commodity investments

According to Carmona (2015), to actually buy a physical commodity itself is one way of investing in commodities and certainly the most traditional one. The most obvious drawbacks of this approach include issues with transportation, delivery, storage and perishability of the commodity. Investing in physical commodities is usually done by hedgers, who look to minimise, if not completely avoid, their financial risks in connection with uncertainties regarding production and delivery of commodities for their businesses (Carmona, 2015). Investing in physical commodities is usually considered problematic, not just owed to their storage costs, but also due to insurance and exchange costs involved (Jensen and Mercer, 2011).

Stocks

Purchasing stocks of a commodity-centric business, according to Carmona (2015), (for example, Shell stocks, in order to invest in oil), is another way to invest in commodities. Some ETFs (exchange traded funds) offer commodity-only portfolios. Even though these are just equity ETFs, they do provide an opportunity to invest in commodities and are therefore known as *commodity ETFs*. It is worth noting that an indirect exposure is attached to this investment, being that shares of natural resource companies are not fully correlated with commodity prices (Carmona, 2015). Lower diversification capabilities of these investments are probably due to strategies that such companies pursue, which – in turn – hedges the exposure to variations in commodity prices (Jensen and Mercer, 2011).

Commodity futures and options

Another way to invest in commodities, according to Carmona (2015), is by investing in commodity futures and options, which is becoming increasingly significant due to transparency and integrity through clearing, and the fact that only minor initial investments are needed in order to ultimately take large positions through leveraging. These products should be structured with a rolling forward of the contracts facing maturity, so that the physical delivery of the commodity can be avoided, as usually large volumes are involved (Carmona, 2010).

Arnott, Chaves, Gunzberg, Hsu and Tsui (2014) argue that trading commodity futures is more interesting for investors than investing in an actual physical commodity, something that is usually interesting only to producers of commodities themselves, since transportation and storage costs are avoided in this manner. When it comes to the commodity futures market, different participants have different motivations and goals. When selling physical commodities, and their price goes down, commercial producers experience a loss. Therefore, in order to hedge this risk, they might sell their commodities upfront at a predefined price, and therefore have a short position in futures contracts. On the other hand, commercial consumers purchase physical commodities from producers and profit less if the prices go up. Therefore, in order to hedge this risk, they buy commodities upfront at a predefined price, and hence have a long position in futures contracts (Arnott, Chaves, Gunzberg, Hsu and Tsui, 2014).

Since supply and demand of the commodity futures market influences the futures' price, commodities futures are traded at a premium or a discount compared to the futures contract with a closer date (Arnott, Chaves, Gunzberg, Hsu and Tsui, 2014). This is known as *contango* or *backwardation*, and it decides the roll yield or returns from selling expiring contracts and purchasing contracts with a later date. For example, gold is in a contango, with a negative roll, as it is characterized by cheap storing. On the other hand, commodities that are more challenging and expensive to store, like energy, will result in more volatile market movements, due to changes between scarcity and surplus of this commodity (Arnott, Chaves, Gunzberg, Hsu and Tsui, 2014).

Additional commodity investment opportunities like exchange traded funds (ETFs), that claim the right to a single commodity, a couple of commodities or even an index following the commodity prices, are becoming more interesting to individual and institutional investors, due to the fact that they are easily assessable, since they are traded on exchanges like other asset classes (Jensen and Mercer, 2011). As of recent, the interest in commodities increased even more due to the abovementioned commodity investment opportunities like ETFs. Aside from commodity ETFs, that have an underlying commodity, there is another commodity investment form – exchange-traded notes (ETNs), that represent a debt instrument with low interest, with whom the investor obtains the exposure to a single commodity or a couple of them. Other commodity investment vehicles that secure commodity exposure by using derivatives on commodity indices, like mutual funds, or by merging a fixed-income asset with an investment in commodities by using structured notes are becoming increasingly attractive (Jensen and Mercer, 2011).

Commodity Exchanges

According to Fabozzi, Füss and Kaiser (2011), there are public marketplace-like specialized exchanges, where the purchasing and selling of commodities take place – with the commodity futures trades happening at a pre-defined price for a pre-defined delivery date. The structure of such exchanges resembles most membership associations, and they have their members‘ benefit as their main focus. Members of the exchange serving as brokers are in charge of transactions (allowed only in form of standardized futures contracts). The trade is reserved for members only. A commodity exchange’s primary role is making sure that there is a well-structured marketplace containing a clear set of rules and standardised contracts (Fabozzi, Füss and Kaiser, 2011).

The origins of a first such exchange, as per Fabozzi, Füss and Kaiser (2011), can be traced back to Osaka, Japan, where farmers used to trade rice futures contracts. However, the first official institution in this regard is the Chicago Board of Trade, in the United States of America, dating back to 1848. The greatest volume of commodity trading takes place here to this day. Another major institution was founded nearly three decades later in London (The British London Metal Exchange, 1877). However, it was not until the foundation of the International Petroleum Exchange (IPE) in London in 1980 that energy futures trading started (Fabozzi, Füss and Kaiser, 2011). Even today, the London Metal Exchange (LME) is considered one of the most significant commodity exchanges, next to the Chicago Mercantile Exchange Group, the Intercontinental Exchange (ICE) and Euronext N. V., according to Soumaré (2022).

When talking about the sheer volume traded, as per Fabozzi, Füss and Kaiser (2011), a comparatively fairly young Chicago Mercantile Exchange (CME), founded in 1998, is considered the biggest futures exchange in the world. There are around 30 significant commodity exchanges globally. In terms of volume traded, most of commodity futures transactions happen in the United States of America and United Kingdom – in addition to Far Eastern powerhouses China and Japan (Fabozzi, Füss and Kaiser, 2011). However, in 2019 emerging markets (like Turkey, India, Russia and China) showed promising increase in commodity exchanges and the Dalian and Shanghai exchanges even outgrew the previously mentioned Intercontinental Exchange (ICE) and the Chicago Mercantile Exchange (CME) when it comes to trade volume (Harasheh, 2021).

The leading world commodity futures exchanges (like the Chicago Board of Trade, London Metal Exchange or Tokyo Commodity Exchange), have a role as third party *clearinghouses*, and these exchanges essentially stand between a single buyer and a seller, making sure that they are free to trade independently, while counterparty risk is eliminated by the exchange itself guaranteeing for every trade that takes place under its watch (Engelke and Yuen, 2011).

2.4. Relevant indices of the commodity market

Since physical commodities are greatly important for the economy, there are commodity indices that underscore trends in the movement of commodity prices (Chambers, Black and Lacey, 2018). However, commodity indices differ greatly from one another, especially when it comes to types of commodities included and the weighting system used. Being that the energy sector is dominant when talking about physical commodity markets, great weight of commodity indices is assigned precisely to this sector, making those indices codependent with the movements on the energy market. Since the weighting system of each index differs, it plays an important role in the entire investment process (Chambers, Black and Lacey, 2018).

According to Carmona (2015), there are three different indices based on their composition: (1) the spot, (2) excess return and (3) total return index (Carmona, 2015). Contract prices are the basis for the calculation of the spot index, whereas the excess return index entails the mentioned spot index, plus the premium earned by the rolling of contracts positions. The third one – the total return index – entails the returns of the abovementioned excess return index, plus the gained interest on completely collateralised contracts' positions of the commodities within the index (Carmona, 2015).

Based on the research done preceding the writing of this thesis, the following indices for commodities markets were identified by the author of the thesis as the most relevant for this purpose and are elaborated on in more detail over the course of this chapter. Additionally, advantages and disadvantages of each index are outlined as well.

2.4.1. Dow Jones Commodity Index (DJCI)

The Dow Jones Commodity Index (DJCI) was created due to the market's need to have a commodity index that consists of liquid commodities without high sector weights and consists mainly of equally weighted commodities from sectors such as agriculture and livestock, energy and metals (S&P Dow Jones Indices, 2021). As opposed to other commodity indices, the DJCI's weighting system is not based on the world production, but on equal weighted commodities. i.e. within each sector, the weighting done is based on the liquidity of the commodity in question, making this index a highly diversified one (S&P Dow Jones Indices, 2021). The DJCI, according to S&P Dow Jones Indices (2021), currently consists of 28 commodities that are selected on an annual basis and there is a spot, excess return and total return option of this index.

2.4.2. Goldman Sachs Commodity Index (S&P GSCI)

One of the most commonly used commodity indices is the Goldman Sachs Commodity Index (S&P GSCI). The Goldman Sachs Commodity Index (S&P GSCI) is an index with a weighting system based on the world production, and consisting of physical commodities with an active and liquid futures market, with an unlimited number of physical commodities included, according to S&P Dow Jones Indices (2021). There is no specific maximum or minimum number of futures defined when it comes to the S&P GSCI, similar to the R/J CRB Index, decribed further below (see Sub-chapter 2.4.4) (Boerse.de, 2021). The Goldman Sachs Commodity Index (S&P GSCI) consists of diverisfied long-term, unleveraged investments in commodity futures (Goldman Sachs, 2021).

The most unique characteristic of this index is its very high weighting of the energy sector, making this its greatest limitation as well. The high share this index has in the energy sector causes a high codependence of the index on the movements on the energy market (Boerse.de, 2021). This is the reason why this index was selected for the empirical analysis of this thesis.

2.4.3. Rogers International Commodity Index (RICI)

Rogers International Commodity Index (RICI), a total return index denominated in USD, was created by James B. Rogers near the end of the 20th century, in order to meet the need for international investment and the index reflects the commodities consumed globally, tracked with future contracts on 38 physical commodities, ranging from the agricultural to the metal and energy sector (Beeland Interests, 2021).

Beeland Interests, Inc. is in charge of the Rogers International Commodity Index, and is supervised by members of the Rogers International Commodity Index Committee. This Committee monitors the Index on a daily basis and its members meets at the end of each year to evaluate the index and evaluate potential changes. One of the characteristics of the RICI is its infrequent composition changes (i.e. its stability), differentiating the RICI from other indices that show significant weights and composition changes on a regular basis (Beeland Interests, 2021).

One of the limitations of this index is the intransparent weighting methodology of the index, next to the poor liquidity of some of itscomponents (Boerse.de, 2021). On the other hand, the RICI is a highly stable index, with broad and consistent components (Beeland Interests, 2021) – which is the main reason why it was selected for the empirical analysis that is part of this thesis.

2.4.4. R/J Commodity Research Bureau Index (CRB)

One of the oldest commodity indices, the Reuters/Jefferies Commodity Research Bureau Index (CRB), considered to be an important early indicator for future inflation or cost development in an industry, was introduced in 1957 and is a good leading indicator of future interest rate developments as well, since commodities tend to respond (even up to six months) faster than bonds (Boerse.de, 2021). However, it is the increased interest in commodities that seems to diminish this quick response. According to Boerse.de (2021), the last change of CRB took place in 2015 and, since then, it contains 19 short-term commodity futures, that are reallocated monthly, so as to keep the index weight constant and prevent the weight of individual commodities becoming too high. One of the main criticisms of this index is due to its high allocation to energy and agriculture sectors, alluding to a rather unbalanced mix of sectors covered by it (Boerse.de, 2021).

3. Modern Portfolio Theory

This chapter aims to give an overview of the basic concepts of the Modern Portfolio Theory (MPT), a theory focusing on the structuring of portfolios so as to maximize the expected return as much as possible, while remaining in line with an investor's respective risk appetite (Fabozzi and Grant, 2001). The points raised in this chapter range from the early beginnings of the portfolio theory creation to the portfolio diversification concepts developed by Harry Markowitz. Moreover, the main advantages and disadvantages of the MPT as well as those of the Capital Market Theory are also taken into consideration.

3.1. Portfolio optimization by Markowitz and the Capital Market Theory

As per Markowitz's paper published in 1952, the decision on what assets to include in a portfolio should not be based solely on their individual expected return and risk, but also on what assets are included in the portfolio overall, with Rubinstein (2002) making an argument that assets that form a certain portfolio should be looked at as a whole rather than individually. According to Fabozzi and Grant (2001), the MPT stresses the importance of including assets with returns that have a lower-than-a-perfectly-positive correlation in a portfolio, in order to lower the risk of the said portfolio and maintain the level of return. This can be achieved by including low – or even, if possible, negative – correlation assets in a portfolio. However, the problem with that is that low to negative correlation assets are quite rare and it is therefore rather challenging to choose the right ones to include in a portfolio, as argued by Fabozzi and Grant (2001). This chapter looks to provide a helping hand when it comes to choosing the right assets to include in an investment portfolio.

3.1.1. Origins of the Portfolio Theory

The concept of portfolio diversification was not first discovered by Harry Markowitz; it was initially introduced by Daniel Bernoulli back in 1738, when Bernoulli argued that risk-averse investors are more likely to diversify, as noted by Rubinstein (2002). After the publishing of Harry Markowitz's doctoral thesis in statistics and its conclusions in the article „Portfolio Selection“ in the Journal of Finance in 1952, the foundation for what would later become known as the MPT had been laid and would even lead to Markowitz becoming a co-recipient of the Nobel prize in the field of economics and corporate finance years later (Mangram, 2013). Even though Markowitz is commonly refered to as the "father" of the MPT, Andrew Donald Roy's significant contribution in the making of the MPT in 1952 must not be overlooked either, even though he became largely inactive in this field of research after this major breakthrough (Markowitz, 1999). Roy argued that investment decisions should be made only after having taken the mean and the variance of the portfolio into account, much like Markowitz himself, but he then went a step further and also took negative investments into consideration and made a recommendation with regard to the choice of a specific portfolio on the efficient frontier (Markowitz, 1999).

In 1958, James Tobin, basing his research on Markowitz's work, developed the *efficient frontier* and the *capital market line* in his "Liquidity Preference as Behavior Toward Risk", implying that investors are not inclined to change their behavior on the market if their expectations remain unchanged, as noted by Mangram (2013). His work focuses on the inverse relationship of the demand for cash and interest rates (Tobin, 1958).

Later on, expanding on the work of Markowitz and Tobin, the theory was developed further by William Sharpe in 1964, John Lintner in 1965 and Jan Mossin in 1966, resulting in the inception of the Capital Asset Pricing Model (CAPM). In summary, although there have been numerous versions and alterations of the MPT over time – from Markowitz and Tobin in the early days to Sharpe, Lintner and Mossin afterwards, the theory remains relevant to this day (Mangram, 2013).

3.1.2. Measuring expected return and risk of a portfolio

According to Fabozzi and Grant (2001), an *efficient portfolio* is the one that brings the investor the maximum expected return while remaining in compliance with the said investor’s risk appetite and the *optimal portfolio* is the one the investor chooses among all available efficient portfolios.

When dealing with risky assets and portfolios, the expected return and risk of a portfolio should be calculated and evaluated. With regard to return, according to Fabozzi and Grant (2001), there are two types: the *ex post* return and the *ex ante* return. The *ex post* return is the actual return of a portfolio over a specific time period and is calculated by adding up the weighted return of each asset, while the *ex ante* return is the anticipated return of a portfolio containing risky assets and is calculated as the sum of weighted average expected returns. The average expected return of an asset is calculated as the sum of all possible rates of return, multiplied by the probability of their outcome (Fabozzi and Grant, 2001).

Markowitz (1952) defined *variance* as the average squared deviation of possible outcomes from the expected value. Therefore, as per Fabozzi and Grant (2001), risk-free assets have a deviation of zero from the expected value. The statistic most commonly used in this regard is the *standard deviation* (square root of variance): the higher the variance and the standard deviation, the riskier the investment (Fabozzi and Grant, 2001).

In order to be able to calculate the risk of a two-asset portfolio, a new measure had to be introduced - the *covariance*. In this instance, the risk is quantified by summing up the weighted variances and adding them to the weighted covariance of the two assets. When measuring the risk in a portfolio containing more than two assets, however, it is necessary to take into account the weighted sum of the level to which returns of all assets change together (Fabozzi and Grant, 2001).

3.1.3. Markowitz's portfolio diversification

The idea of portfolio diversification, first challenged by Markowitz (1952), assumed that if an investor were to diversify his portfolio by investing a part of his money in shares of one company and a part in shares of another, given that these two individual portfolios have the same variance, a portfolio combining them would have a lower variance. Before the creation of the portfolio theory – although diversification as such was already discussed at that time – there were no quantitative measures for portfolio diversification, meaning that diversification benefits could not yet be properly utilized (Fabozzi and Grant, 2001).

Markowitz's portfolio theory is often called the *two-parameter model*, being that the two parameters, expected return and risk, are used in the investment decision-making process (Fabozzi and Grant, 2001). The MPT is widely considered the standard theoretical model used for constructing an optimal portfolio and asset allocation, as well as providing a springboard for further improvements like the Capital Asset Pricing Model (CAPM) or the differentiation between systematic risk and diversifiable risk (Michaud, 1989).

Implementing diversification in the way Markowitz suggested is linked to creating portfolios with the highest expected return with a given variance, the so-called *Markowitz efficient portfolios* or just *efficient portfolios*. In order to create such a portfolio, a few assumptions (as decribed later on in this chapter) had to be laid out first (Fabozzi and Grant, 2001).

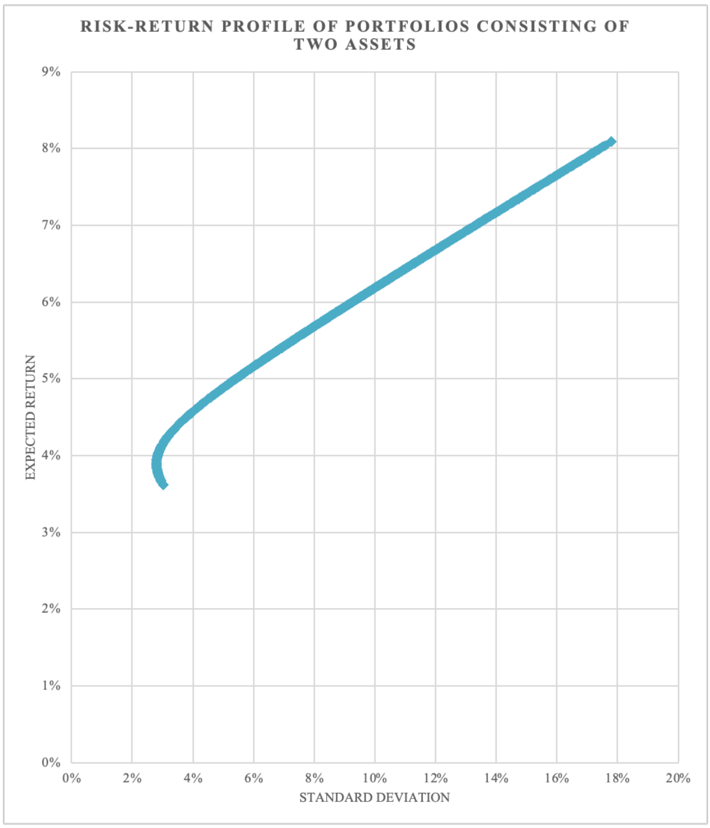
The Modern Portfolio Theory, developed by Markowitz, is built on the assumption of efficient markets, i.e. that investors search for the best possible return, in line with their respective risk appetites, and that the trading price of assets is the best estimate of their future value, since all investors are provided with the same investment information (Bank Investment Consultant, 2006).

The MPT is based on a few assumptions – like the one that all investors seek efficient portfolios (i.e. all investors are *rational*); that borrowing money at a risk-free rate (for example the 3-month U.S. treasury bills) is possible for every investor; that investors share the same risk-return anticipations and are risk-averse (desire for higher expected return at equal risk levels, as well as lower risk for equal expected return levels); that the inflation and the interest rate environment remain unchanged (i.e. capital markets in equilibrium), as outlined by Bank Investment Consultant (2006). Further assumptions regarding capital markets, as noted by Füss, Adams, Tilmes, Glück and Lenz (2011), include the assumption of a perfect market, implying that there is an unlimited division of securities, no transaction cost, no taxes, no cost of obtaining information and no market entry barriers. Furthermore, only linear correlations are being measured, virtually ignoring the non-linear ones. Additionally, the data used in the MPT is the projection of future returns based on historical data, which poses the question of accuracy of the said projection of expected returns for the future (Füss, Adams, Tilmes, Glück and Lenz, 2011). Lastly, as outlined by Fabozzi and Grant (2001), the MPT is based on the asumption that all investors share the same expectations when it comes to the expected return, risk and covariances for all assets (i.e. the assumption of homogeneous expectations) and that they all work with(in) the same investment period as well.

A differentiation between a *feasible* and an *efficient* portfolio should be made at this point. According to Fabozzi and Grant (2001), a feasible portfolio is a portfolio that can be created with the available assets, while a feasible set of portfolios includes all portfolios on the curve on the expected return-standard deviation graph and they represent a group of all feasible portfolios. Being that Figure 1 represents a portfolio consisting of two assets, the feasible set of portfolios is shaped like a curve and all portfolios on it a feasible set of portfolios (Fabozzi and Grant, 2001).

Figure 1

Risk-return profile of portfolios consisting of two assets

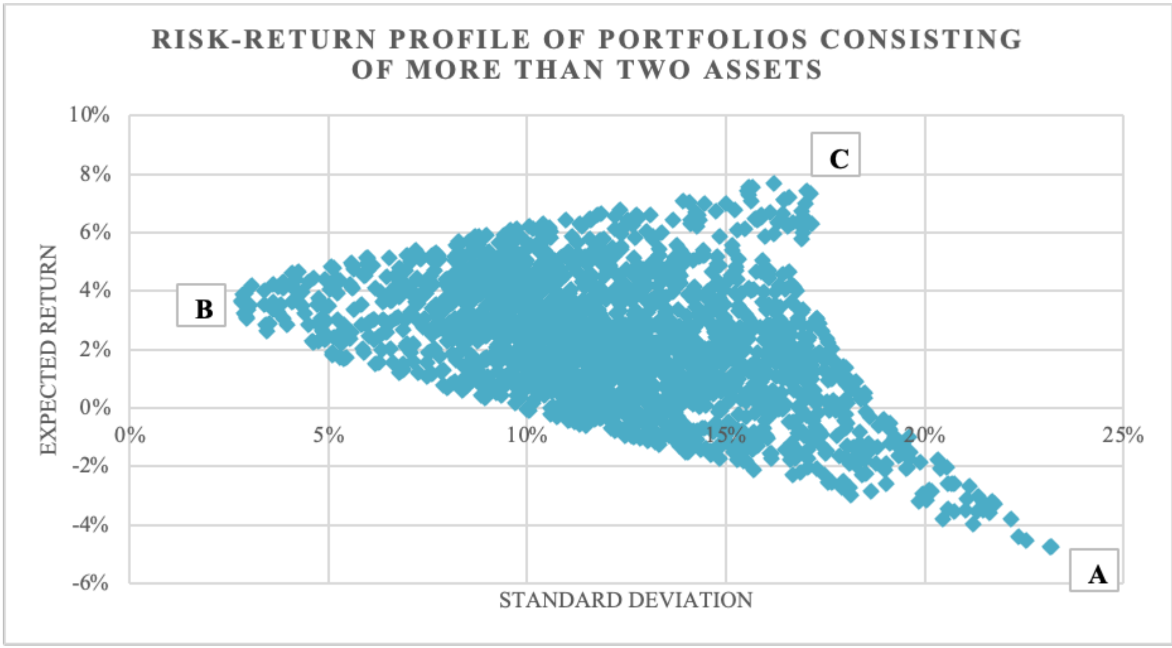


Note. Source: Own calculation.

However, if a portfolio of more than two assets is created, a feasible set of portfolios is no longer a curve, as it takes up an entire area, as seen in Figure 2 (Fabozzi and Grant, 2001).

Figure 2

Risk-return profile of portfolios consisting of more than two assets



Note. Source: Own calculation.

According to Fabozzi and Grant (2001), the one porfolio out of all feasible portfolios with the highest expected return and the same level of risk is considered the *Markowitz efficient portfolio* or the *mean-variance efficient portfolio.* Portfolios located higher than this *Markowitz efficient portfolio* on the expected return-standard deviation graph (Portfolios B to C) are considered the Markowitz efficient set of portfolios (Fabozzi and Grant, 2001).

Keeping in mind the abovementioned MPT assumptions, as per Schulmerich (2013), an efficient portfolio as according to the MPT can be selected as well, with an efficient portfolio being the one with the greatest expected return and the lowest volatility. The curved line on the expected return-standard deviation graph formed by connecting every expected return to its lowest volatility and creating a set of efficient portfolios, consists of two parts: the less beneficial and the more beneficial part of the line. The *efficient frontier*, is more beneficial for the investor due to its higher expected returns and is therefore positioned above the MVP. The MVP (minimum variance portfolio) is the part of the curve with the lowest volatility (Schulmerich, 2013). Any portfolio above the efficient frontier (also known as the *Markowitz efficient frontier*) can not be obtained, while any portfolio below it is inferior to portfolios on the Markowitz efficient frontier (Fabozzi and Grant, 2001). Investment consultants decide whether a portfolio is efficient or not with the help of a mean-variance analysis, which takes into account the expected return, risk and covariances of single assets, in order to analyse the risk-return profile of the portfolio (Bank Investment Consultant, 2006). A lot of different factors speak for using the mean-variance analysis instead of a utility analysis (Markowitz, 1991), including the mean-variance analysis being more cost-effective, feasible and easier to conduct.

The question of which portfolio on the Markowitz efficient frontier to choose does not remain unanswered. The optimal portfolio, according to Fabozzi and Grant (2001), is the one that should be selected and this portfolio depends on the risk and return preferences of the investor, i.e the utility function. Therefore, the optimal portfolio, the best choice on the Markowitz efficient frontier, is the one where the utility function touches the Markowitz efficient frontier (in the point P_{MEF}) and since the utility function is linked to investors' preferences, different preferences may lead to different utility functions and – as a result – to a different optimal portfolio altogether (Fabozzi and Grant, 2001).

As per Fabozzi and Grant (2001), problems might occur when trying to quantify this utility function, as no instructions on how to measure one exactly even exist and it is therefore up to investors to decide which portfolio on the Markowitz efficient frontier is the optimal portfolio for them, given their individual risk and return preferences.

Portfolio optimization with commodities

In an empirical research conducted by Fabozzi, Füss and Kaiser (2011), it was analysed if adding commodities to a portfolio of stocks (both U.S. and international) and bonds (with treasury bills representing the risk-free assets) could lead to diversification benefits. Efficient portfolios, observed among a group of efficient portfolios with remarkable risk-return characteristics, lie on the line created by the group of porfolios between the minimum variance portfolio (MVP) and the maximum return portfolio (MaxEP), as according to Markowitz's theory (Fabozzi, Füss and Kaiser, 2011).

As outlined by Fabozzi, Füss and Kaiser (2011), by adding commodities to a portfolio of stocks and bonds, the efficient frontier moves around the minimum variance portfolio (i.e the treasury bill rate) in a counterclockwise fashion. This movement of the efficient frontier implies that increased risk-adjusted returns could be gained by this portfolio (Fabozzi, Füss and Kaiser, 2011).

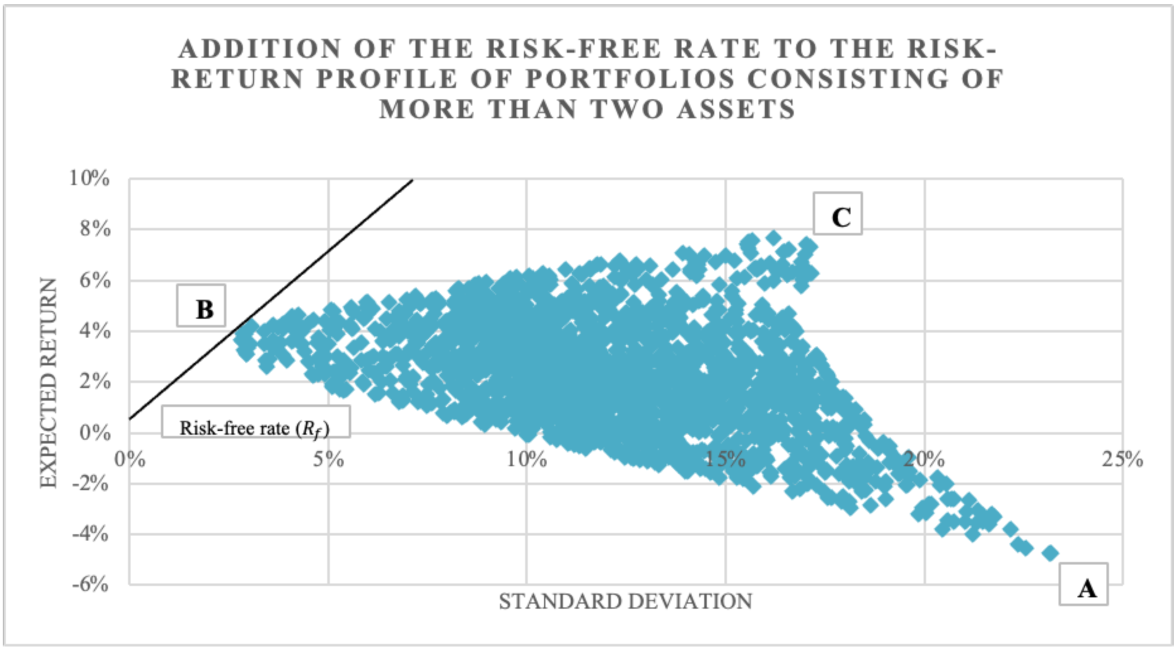
3.1.4. Capital Market Theory

When not considering the risk-free rate, the optimal portfolio is the one where the utility function touches the efficient froniter, according to the MPT and as already outlined in Sub-chapter 3.1.3. However, when the risk-free rate is taken into account, and the assumption is that investors are able to borrow and lend at a risk-free rate, that is an entirely different conversation (Fabozzi and Grant, 2001).

Therefore, the abovementioned efficient frontier assumed that lending money or depositing it with a risk-free rate (R_f) and without volatility is not possible. However, this option is taken into account by Schulmerich (2013). Since there are numerous portfolio possibilities on the efficient frontier, the addition of a risk-free rate (R_f) helps choose the right portfolio on the frontier, as outlined by Schulmerich (2013) (see Figure 3).

Figure 3

Portfolios consisting of more than two assets including risk-free rate of return



Note. Source: Own calculation.

In this case, the portfolio that should be chosen is the *tangency portfolio* (i.e. market portfolio), is the one where the steepest line touches the efficient frontier (Schulmerich, 2013). This "steepest line" is called the *Capital Market Line* (CML) by one of the theory's creators, William Sharpe (Fabozzi and Grant, 2001). The CML is defined as the excess of the expected return over the risk-free return divided by the risk, as per Fabozzi and Grant (2001). Therefore, it serves to maximize the Sharpe ratio, being that the Sharpe ratio represent the excess of portfolio return over the risk-free rate (R_f), divided by portfolio volatility (Schulmerich, 2013). The CML is also called the *equilibrium market price of risk*, since it shows the additional return that needs to be earned in order to reimburse the investor for taking on the additional risk, as noted by Fabozzi and Grant (2001).

As argued by Hull (2014), the steepest line's relation between the expected return and the volatility is linear. The tangency portfolio represents a portfolio of all risky investments. If it is assumed that an asset is not a part of the tangency portfolio, no investor would be interested in it, which would lead to a price decrease and an increase in expected return, which would in turn make it a part of the tangency portfolio. This is the reason why a tangency portfolio is referred to as the *market portfolio*. In order to maintain a balance between demand and supply, it is necessary that the price of every risky investment is levelled in a way that the share of such an investment in the tangency (or market) portfolio corresponds to the share of this investment on the market (Hull, 2014).

3.2. Advantages and disadvantages of the Modern Portfolio Theory

Muralidhar (2015) argues that the MPT might not be as problematic as some other authors suggest, as it provides an overall view on the investors' behavior, with an underlying assumption that investors are risk-averse and value wealth. However, it is precisely the main assumptions of MPT that might be considered the theory's greatest drawback, being that it is a model for risk-averse investors who value wealth and can therefore not be the ideal choice for a different type of investor (Muralidhar, 2015).

A number of authors see a list of limitations concerning the MPT, especially in the context of today's rapidly changing market conditions and fast information flow. As per Bank Investment Consultant (2006), the MPT states that the expected return is only linked to market-related risks. Therefore, non-market risks like economic or company-specific risks are not linked to a higher expected return. According to the MPT, the *alpha*, a manager's exceeding return over the benchmark, amounts to zero in the long run, even though random non-market changes might lead the alpha to differ from zero over a specific period od time. However, if this were really true, investment managers would hardly be in high demand – and yet they are, as they aim to have positive alphas and achieve higher returns than the market while taking on fewer risks (Bank Investment Consultant, 2006).

Schulmerich (2013) recognizes two main limitations of the MPT. Firstly, investors, both retail and institutional ones, use more asymmetrical risk measures (like Value at Risk), since they are known to have a more practical application, especially after the crises of 2008 and 2011, particularly in comparison to symmetrical risk measures that are not as successful in measuring risk in times of turmoil. Secondly, the focus of the MPT is on the absolute risk and return of a porfolio, which is not of the utmost importance to institutional investors, that aim at exceeding the benchmark's performance by remaining within the boundaries of the given tracking error budget instead of prioritising the absolute portfolio return (Schulmerich, 2013).

Warner (2010) argues that the MPT falters every few years, due to the implementation of the theory, rather than the theory itself. Namely, he argues the MPT did not account for the market changes that took place in the 21st century, such as the 1-2% changes in index movement on a daily basis, which was a change that would historically occur over the course of months and not within a single day. Also, as per MPT, the market crises of 1987, 2000 and 2008 are unusual one-off events that would not have a negative impact on a long-term diversified portfolio, as argued by Warner (2010). However, investors that have invested in treasury bonds in the time period from 1969 to 2009 would have gained higher returns than those investing in an S&P 500 fund within that same timeframe (Warner, 2010).

One of the biggest criticisms of the MPT is that upside and downside volatility are both considered a risk, as a movement away from the mean on the bell-shaped normal distribution curve is linked to standard deviation (Warner, 2010). However, he argues that upside volatility is not perceived as being a risk by an investor, since investors try to reduce their losses rather than their risk.

Fabozzi and Grant (2001) too argue that one of the limitations of the MPT is that the theory takes into account both returns, above and below the expected one, since it is defined as a deviation of possible outcomes from the expected value. It has been suggested to exclude the returns above the expected return when measuring risk, as those are actually desirable from the investors' point of view (Fabozzi and Grant, 2001). Therefore, they argue that a measure of risk that takes into account just the returns below the expected return was introduced – the so-called *semi-variance.* In case of an asymmetrical probability distribution, variance faces its limitation in being the only measure of risk and therefore *skewness* needs to be used along with it, as noted by Fabozzi and Grant (2001).

Furthermore, MPT does not consider the serial correlation between assets, especially alternative assets, and looks at every period of investment independently (Warner, 2010). However, he argues that returns of the current month often imitate the returns of previous months and should therefore be corrected in order to have the correct volatility of an asset. One additional criticism of the MPT is that it assumes that all investors are rational, which could not be further from the truth in cases of extreme upswings and declines on the market, when investors do not behave rationally at all (Warner, 2010).

Davidow (2020) identifies a few constraints of the MPT as well, one of them being the MPT's assumption of rational investors, similar to Warner (2010). The MPT assumes that investors are rational, however this is not always the case, in particular not in bull markets, where investors seek higher returns (Davidow, 2020). Thus, it is assumed that investors choose the best possible portfolio and not the portfolio with the highest return. Furthermore, the historical data used in the MPT is a good indicator of future movements but it should not be the only source used for predicting future behavior on the market (Davidow, 2020).

As argued by Proelss and Schweizer (2011), Markowitz's normal return distribution hypothesis did not hold true in case of the majority of commodities analysed, since standard deviation was used as the only measure of risk in portfolios of different commodities. However, they argue the standard deviation is not considered an ideal measure of risk for a portfolio of commodities. Being that it tends to underestimate the higher risk linked to commodity investment (Proelss and Schweizer, 2011), the conditional Value at Risk (CVaR) is thought to be a better choice for risk measurement in a portfolio consisting of commodities.

Additionally, it should be noted that markets are not always efficient and constant risk, returns or correlations over longer periods of time are not always the case, as outlined by Davidow (2020). Especially correlations, as one of the main selling points of the MPT, are constanly increasing over the years and are therefore not constant at all, which can be traced back to the higher connectivity of markets in the recent years and the lowering of interest rates by central banks worldwide. Negative and low correlation was more frequent in the time period from 1999 to 2008 than from 2009 to 2018. In periods of great turmoils, like in 2008 or in the fourth quarter of 2018, there was a noticeable increase of correlation between the main asset classes (Davidow, 2020).

As outlined by Davidow (2020), the interest in environmental, social and governance investing increased over the last years, shifting the focus from performance-based investing to investing into assets that are in line with investors' interest and passions. This influence of investors' personal goals and preferences on the portfolio is just another additional aspect not considered in the MPT that has gained in relevance over the last years (Davidow, 2020). One of the criticisms of the MPT is that it is way too exclusive and limited in assuming all projections with a sole market factor and failing to consider other relevant factors like market capitalization, economic growth or inflation (Muralidhar, 2015).

As outlined by Rice (2017) as well, the MPT fails to acknowledge the changes in markets and economies or to account for geopolitical changes. He argues that central banks and government policies nowadays have a greater influence on asset prices than the classical supply and demand or business cycle changes. Rice (2017) recognizes unstable correlations as one of the main limitations of the MPT, much like Davidow (2020).

According to the MPT, the results form a bell-shaped curve (normal distribution), but due to the rise of digital economy that has a different trend when compared to the rest of the economy, this bell-shaped curve started to look more like a *Pareto curve* (Rice, 2017). The reason for this lies in the insignificant production and distribution cost of this type of industry, as argued by Rice (2017).

Globalisation also had an influence on the significance of the MPT, as argued by Rice (2017). When the MPT was first created, the most significant part of the world's GDP was assigned to the USA, making it dominant on capital markets and not significantly impacted by events taking place elsewhere in the world. However, this changes with globalisation, resulting in a decline of USA's share in the global GDP and about a half of the income of the S&P coming from abroad, as outlined by Rice (2017). On the other hand, contrasting the declining influence of domestic markets, influences of great foreign countries, such as China, on the global markets are becoming very significant and unpredictable and, as stated by Rice (2017), the MPT is not equipped to make predictions in such environments.

As per Davidow (2017), the MPT did not lose its relevance, but it does need to adapt to the changing market conditions. He argues that changes in the market, not only due to globalisation and the interconnectivity of the markets, but also due to higher volatilities and lower returns on bonds and equity, are some of the new factors that were not considered back in the day when MPT was first created and will surely continue to be a topic of discussion over the next years. The effects of globalisation on the stock markets can be best observed by taking a look at how the interconnected markets jointly reacted to events like the Brexit vote, the decline in the economy of China or the USA elections (Davidow, 2017). Since information nowadays travels very fast and the responses on the markets to certain information are uncomparably faster than in the past, higher volatility on markets is almost a given. Bond returns have been low since the 2008 crisis and the equity markets - due to their historically low rates - are not likely to increase significantly in the future either (Davidow, 2017). As stated by Davidow (2017), one of the possibilities to work around the aforementioned factors may be to invest in non-traditional assets like commodities and by responding to changes in the markets swiftly and with a flexible tactical asset allocation, in addition to increasing the global diversification.

4. Overview of empirical research

This chapter aims to provide a summary of the selection of empirical research chosen for the writing of this thesis and the main takeaways from it. A number of scientific articles was carefully analysed in order to get a better idea when it comes to investing in commodities, the nature of their correlation as well as the diversification benefits associated with investing in this specific asset class. It is also an important goal of this chapter to summarize the main conclusions of the empirical research it focuses on and provide a clear understanding of all advantages and disadvantages that investing in commodities may bring.

4.1. Summary of selected empirical research

The empirical research in this thesis consists of ten articles selected for this purpose based on their content and relevance in the field of commodity investing. The topics of the selected empirical research papers range from the analysis of the addition of commodities to an efficient portfolio; correlation and diversification analysis of bonds, stocks and commodities; influences of finalisation of commodity futures market to hedging possibilities of various commodities. These various topics are therefore summarized unter the sub-chapters addressing different impacts on commodity markets and the interaction of commodities with other asset classes, the diversification benefits of commodities and the risk reduction capabilities per commodity. Publishing dates range from 2011 to 2019.

4.1.1. Different impacts on commodity markets and interaction of commodities with other asset classes

The impact of financialization of the commodity markets is analysed in Main, Irwin, Sanders and Smith's (2018) article, outlining the rapid increase of alternative investments in the mid-2000s, as well as the investment in commodity futures and overall commodity prices in that time period. The goal of this article is to analyse whether the risk premium of long-term commodity investors is affected by the financialization of commodity futures markets. A *cost-of-carry* model for storable commodity prices is used to illustrate the duration of the impact of return on risk premium. Daily futures prices for 19 commodity futures markets in the time period from 1961 to 2014 serve as data for the analysis of whether the risk premium is influenced by the financialization of commodity futures markets. A decrease in the risk premium of energy future prices is noted after 2004 (with 2004 being the year in which the financialization of commodity markets began). However, during the same time period, there was a price increase in most non-energy futures prices (Main, Irwin, Sanders and Smith, 2018).

The cost-of-carry model for storable commodity prices is used in Main, Irwin, Sanders and Smith's (2018) research in order to analyse changes in returns of commodity futures markets. The data used in the research includes 19 commodity futures markets in the time period from 1961 to 2014, including the New York Mercantile Exchange (NYME) energy market, metals market, Intercontinental Exchange Softs Market, Chicago Board of Trade (CBOT) grain market and Kansas City Board of Trade (KCBT) wheat market. Since the cost-of-carry model is relevant for storable commodities, the data sample is narrowed down to storable commodities, which is why livestock is not a part of the data sample (Main, Irwin, Sanders and Smith, 2018).

According to Main, Irwin, Sanders and Smith (2018), firstly, changes in spot prices, changes in future returns and changes in return components from 1990 to 2014 were analysed. The results show that the ex post spot premium of energy markets for nearby futures after 2004 was in significant decline, with especially the WTI crude oil decreasing from an average premium of 9.5% (from 1990 to 2004) to -5% annually (from 2005 to 2014). However, the results of the non-energy markets (like metal, soft commodities and grains) significantly differ from the energy markets, in that they do not show a constant decrease in risk premium. Therefore, a systematic decrease of market risk premiums in all energy and non-energy markets could not be proven. The average risk premiums for all 19 markets contained in the sample in the time span of 25 years was an annual decrease of 0.4% (Main, Irwin, Sanders and Smith, 2018).

A market decrease for ex post premiums after 2004 was noted for deferred futures and energy markets as well, with especially the WTI crude oil decreasing from an average of 9.4% (from 1990 to 2004) to 1.3% annually (from 2005 to 2014). The average risk premium for all 19 markets increased from 1.2% (from 1990 to 2004) to 2.1% annually (from 2005 to 2014). There is some proof that risk premiums in all 19 commodity futures markets decreased systematically from 2005 onwards, due to the increased average risk premium in deferred futures markets. Additionally, an analysis of the ex post premium of the 19 commodity futures markets per decade was conducted, showing that increasing risk premiums in the energy, metal, soft commodity and grains futures markets were observed for two decades, indicating no significant impact of the financialization (Main, Irwin, Sanders and Smith, 2018).

Therefore, in summary, the average level of unconditional risk premiums was mainly not impacted by the finacialization of the commodity futures market in the mid-2000s, implying that returns in commodity futures markets are primarily driven by individual random supply and demand fluctuations (Main, Irwin, Sanders and Smith, 2018).

Concerning the topic of interaction of commodities with other asset classes, Narayan, Thuraisamy and Wagner (2017) argued – in their article with focus on the delayed interaction of bonds, equity, gold and oil markets in the time period from January 1950 to June 2015 – that financial dependence based on returns or cycles was not a topic of discussion in literature for a long time. This article aims to reveal the interaction between financial markets, commodities and inflation, in order to improve the understanding of cross-market asset pricing relations by focusing on U.S. markets and identifying cycle components, while their research focuses on the returns and cyclic components of price levels.

Data used in this empirical research includes U.S. bonds (the performance of a rolling investment in 10-year treasury bonds), equity market (Standard and Poor's 500 index returns adjusted for dividends) and commodity data (gold and oil prices, the gold and oil investment performance in USD). The U.S. Consumer Price Index (CPI) for urban consumers is used to measure the price level series (Narayan, Thuraisamy and Wagner, 2017).

The macroeconomic uncertainty measured by equity market volatility has been taken into account as well by using the U.S. equity market realized volatility. Based on daily returns of the S&P's 500 index, a series of monthly realized equity market volatilities has been calculated in order to measure the abovementioned macroeconomic uncertainty, as outlined by Narayan, Thuraisamy and Wagner (2017).

The statistical results of the continually compounded returns and the changes in the cycle components are analysed and show that the highest volatility is noted in the case of oil returns, closely followed by gold, equity, and bond returns. Negative skewness was observed in equity returns, while oil returns show positive skewness. The observed kurtosis was the highest for oil and gold returns (Narayan, Thuraisamy and Wagner, 2017).

A weak linear dependency has been noted between bond and stock returns (a correlation of 9%). The interaction between bonds and equity therefore has a self-enforcing and diminishing dynamic component, since bonds significantly and positively Granger cause equities while at the same time significantly and negatively Granger causing bonds. The research suggests that shocks to uncertainty impact the equity market as well as the bond market, being that positive shocks to volatility yield a delayed decrease in equities, accompanied by an increase in bond prices – and vice versa (Narayan,Thuraisamy and Wagner, 2017).

A negative relation between delayed bond returns and gold is highlighted by Narayan, Thuraisamy and Wagner (2017). While examining commodities and inflation, it was concluded that bond returns negatively Granger cause oil prices, i.e. high bond returns predict lower oil prices and vice versa. Thus, it was noted that oil prices positively Granger cause inflation. However, it was also discovered that a shock to inflation does not Granger cause any observed asset class. Since it was proven that there is a correlation between CPI and oil (22%) and gold and CPI (12%) and, on the other hand, a negative correlation of bonds and stocks, this suggested that commodities, oil and gold respond relatively fast to any CPI changes (Narayan, Thuraisamy and Wagner, 2017).

The empirical research conducted by Narayan, Thuraisamy and Wagner (2017) highlights a delayed cross-market pricing transmission from gold to bonds and – consequently – to oil and inflation. There are two cross-market pricing effects brought up in this article:

- The interaction between bonds and equity, with a self-enforcing and diminishing dynamic component;

- Macroeconomic uncertainty affecting the equity, then the bond market and consequently back to the measure of uncertainty.

Another research, by Öztek and Öcal (2017), also deals with the topic of commodity interaction with other asset classes and focuses on correlations of commodity markets during the financialization and in the wake of the latest financial crisis. The article aims to spotlight a rising trend in correlations focusing on two commodity sub-indices: the agricultural commodity sub-index (S&P AG) and the precious metal commodity sub-index (S&P PM). With regard to the agricultural commodity sub-index, recent discoveries suggest that the cause of the high correlation lies in the high market volatility in times of financial crises. When it comes to the precious metal commodity sub-index, the market volatility plays an important role in the dynamic nature of correlation (Öztek and Öcal, 2017).

Data used in this empirical research consists of daily price series of agricultural commodity and precious metal sub-indices of S&P GSCI and the S&P 500 index from the Global Financial Data in the time period 1990-2012. Weekly return rates, derived from logarithmic differencing Thursday closing prices, are used in this research, so as to avoid possible end-of-week effects (Öztek and Öcal, 2017).

According to Öztek and Öcal (2017), the conditional correlations between the commodity and stock market indices are displayed in the form of multivariate GARCH models with time-varying conditional correlations. In order to define the conditional correlation equation, the smooth transition conditional correlation (STCC) model and the double smooth transition conditional correlation (DSTCC) model are used, due to their high flexibility. The two mentioned models, the STCC and the DSTCC, provide researchers with a chance to characterize the increasing trend by using time as a variable.

The normalized price series of indices from the year 1990 up until June 2002 is observed in the research. A higher performance of the S&P 500 indices (the stock market), is observed between 1990 and 2000. The research shows that commodity sub-indices, particularly the S&P PM, have enjoyed a higher performance than the stock market indices since mid-2000s (Öztek and Öcal, 2017).

Within the examined period of time, the S&P PM has had the highest mean return rate, followed closely by S&P 500. An inverse relationship between risk and return was noted in the time period in question: the highest volatility level with the lowest mean return (S&P AG) and the lowest volatility with the highest mean return level (S&P PM). The S&P PM index is skewed to the right, suggesting that significant negative returns are not as likely as significant positive returns. This leads to the conclusion that the S&P PM is not riskier with regard to losses. The S&P AG and S&P 500 indices are skewed to the left, which is common for most time series – also meaning that excess kurtosis makes it more likely for all indices detecting extreme returns to occur (Öztek and Öcal, 2017).

When observing the unconditional sample correlations between indices, the correlation is almost zero between the S&P PM and the S&P 500 and very low between the S&P 500 and the S&P AG, meaning that agricultural and precious metal commodity indices could be a good opportunity for portfolio diversification and risk reduction (Öztek and Öcal, 2017).

According to the empirical research conducted, the most statistically significant variable influencing correlations of both commodity sub-indices with the stock market index is the time variable – a variable most suitable for conditional correlations of said indices (Öztek and Öcal, 2017).

With regard to the pairing of the S&P PM and the S&P 500, research results suggest that a conditional correlation upwards shift occurred in November 2003, much before the last of financial crises, and has ever since variated between zero and −0.28 as per magnitude of the shock to the stock market index. Therefore, it can be concluded that the precious metal market offered good portfolio diversification chances up until November 2003. After the mentioned increase and substantial negative shocks in November 2003, correlation increases were not so significant, making precious metals a great diversification source for a portfolio (Öztek and Öcal, 2017).

With regard to the pairing of the S&P AG and the S&P 500, the correlation had oscillated between −0.03 and 0.19 over quiet periods, whereas there was a fluctuation anywhere from 0.11 to 0.58 in volatile ones. When both markets were quiet, the correlation was at −0.03. With the rise of volatility of the agricultural commodity market, the correlation changed to 0.19. However, when volatility of the stock market increased, the correlation rised to 0.11. If both markets were volatile, which usually happens in times of financial crises, the correlation was at 0.58 (Öztek and Öcal, 2017).

Throughout financial crises, the risk tolerance of financial investors decreases, as they abandon their long positions in commodity markets (Öztek and Öcal, 2017). Therefore, as volatility of financial markets increases, the correlations of commodity markets with other financial markets increase as well. Nevertheless, the role of information concerning commodity futures can be seen as an explanation for the positive interconnection between volatility of the agricultural market and its correlation with the stock market, since commodity futures markets send negative signals about the condition of the global economy in turbulant periods, which might influence the behavior of financial investors in a negative way (Öztek and Öcal, 2017).

The conditional correlation between the S&P AG and the S&P 500 has an upward trend, being close to zero up until August 2008 before increasing to 0.37. The findings of this research indicate that the volatility measures of markets are quite important when analysing the correlation structure of the agricultural commodity sub-index with the stock market. The influence of the stock market volatility on correlation is analysed as well. Quieter periods lead to modest conditional correlation (from −0.03 to 0.11) unlike more turbulent periods, known for a stronger conditional correlation (from 0.19 to 0.58). These high levels of correlation – such as 0.58 – can therefore be linked with high volatility during periods of financial crises (Öztek and Öcal, 2017).

A hypothetical portfolio, including two assets (S&P 500) and one of the abovementioned commodity sub-indices (S&P AG or S&P PM), was created by using actual out-of-sample weekly data from January to December 2013. It was discovered that the demand for commodity indices relates to risk-aversion of investors (low demand by less risk-averse investors). The demand increases with the degree of risk-aversion. Furthermore, the estimated model proposed that portfolios offered higher gains during quieter periods, as opposed to more volatile ones, known for generating no gain (Öztek and Öcal, 2017).

This article aims to spotlight the impact of return correlations between commodity and stock markets and the effect financial crises have on such correlations. In addition, the evolution of correlations during the financialization of commodity markets has been observed as well, resulting in following conclusions (Öztek and Öcal, 2017):

- The hypothesis of an upward trend between the correlation of the agricultural commodity sub-index (S&P AG) and the stock market index (S&P 500) was not confirmed. The increase in correlation between the two is mostly due to financial crises, complemented by relatively high market volatilities. Thus, the agricultural commodity market is regarded as a better opportunity for portfolio diversification in the midst of more quiet periods.
- Although an increasing trend between the precious metal sub-index (S&P PM) and the stock market index (S&P 500) is noted, correlation levels are highly affected by market volatilities throughout financial crises. Since their highest recorded correlation level was at 0.20, it is considered very low in comparison to correlation levels between other markets, indicating substantial benefits for portfolio diversification.
- The research suggests that high gains arise from portfolio diversification between commodity and stock markets as opposed to investing only in the stock market. Thus, according to the research in question, the portfolio provides better improvements over more quiet periods rather than over more volatile ones.
- Additionally, the research indicates that markets are not integrated enough to have a lasting trend in the correlation and that they are rather dependent on market volatility. Therefore, the optimal weights of the assets in the portfolio should be tailored in accordance with the market regimes.

When it comes to the impact of financialization of the commodity markets, as analysed in Main, Irwin, Sanders and Smith's (2018) research, the risk premium was mainly not impacted by the finacialization of the commodity futures market in the mid-2000s, implying that returns in commodity futures markets are primarily driven by individual random supply and demand fluctuations. While analysing the interaction of the commodity market with other asset classes, a delayed cross-market pricing transmission from gold to bonds and – consequently – oil and inflation, was outlined in the empirical research conducted by Narayan, Thuraisamy and Wagner (2017). Additionally, Öztek and Öcal's (2017) empirical research aims to spotlight the impact of return correlations between commodity and stock markets and the effect financial crises have on such correlations, suggesting that high gains arise from portfolio diversification between commodity and stock markets as opposed to investing only in the stock market.

4.1.2. Diversification benefits of commodities

The article written by Cotter, Eyiah-Donkor and Potì (2017) focuses on whether a portfolio of stocks, bonds and treasury bills of a risk-averse investor can be improved in terms of risk and return via means of adding commodities and currencies, with the implementation of both the dynamic unconditionally efficient portfolio strategy and the conditionally efficient mean-variance strategy.

The data used in Cotter, Eyiah-Donkor and Potì's (2017) research includes monthly log returns of stocks, bonds, treasury bills, as well as commodity and currency futures. The in-sample spanning tests conducted indicate that commodities and currencies could offer diversification benefits – the commodity boom period (2000-2014), in which commodities did not provide diversification benefits, notwithstanding (Cotter, Eyiah-Donkor and Potì, 2017). However, these diversification benefits are very close to the minimum-variance portion of the mean-variance frontier, making them uninteresting for investors in either case. Additional in-sample research suggests that there are benefits arising from investing in commodities and currencies, as they lead to higher Sharpe ratios. On the other hand, out-of-sample research indicates that no diversification benefits are to be seen. Cotter, Eyiah-Donkor and Potì's (2017) research indicates that the in-sample proof of diversification may be false, as it assumes a flawless forecast for returns, variances and covariances and financialization might weaken diversification capabilities.

This empirical research contains an in-sample analysis of the advantages of a commodities and currencies investment, conducted via means of a mean-variance spanning test. This part of the research shows that adding commodities and currencies to a portfolio may very well improve the risk return profile, however rather near the global minimum variance share of the frontier. These results may also be interpreted as the two markets (commodity and financial) becoming more assimilated, resulting in a reduction of diversification capabilities (Cotter, Eyiah-Donkor and Potì, 2017).

Additionally, the predictability of asset returns in Cotter, Eyiah-Donkor and Potì's (2017) article was tested by predicting a seven variable VaR for stocks, bonds, commodities and currencies, each including delayed values of returns of these four asset classes and three additional forecast variables. The underlying idea was that it is not known initially which variables have predictive capabilities for forecasting the returns of assets.

Thereafter, Cotter, Eyiah-Donkor and Potì (2017) analyse whether advantages of predictability can be used to develop *ex post* portfolio performance strategies. The results were calculated for four different portfolios, i.e. panels containing different asset classes, with Panel A including stocks, bonds, treasury bills, commodities and currencies; Panel B including stocks, bonds, treasury bills and currencies; Panel C including stocks, bonds, treasury bills and commodities and Panel D including stocks, bonds and treasury bills (Cotter, Eyiah-Donkor and Potì, 2017).

The comparison between different portfolios or panels confirms the findings of spanning tests conducted in the first part of Cotter, Eyiah-Donkor and Potì's (2017) research. The research indicates that a mean-variance investor would achieve a better risk-return profile of a portfolio by adding currencies or commodities to it. A much better portfolio performance would, however, be achieved by adding both currencies and commodities to a traditional portfolio (Cotter, Eyiah-Donkor and Potì, 2017).

Whether diversification capabilities of commodities and currencies can be achieved in an out-of-sample analysis was observed as well, via standard performance metrics (Cotter, Eyiah-Donkor and Potì, 2017). It was concluded that traditional portfolios have higher Sharpe ratios than the ones including commodities and currencies. These results of the Sharpe ratio observation were confirmed by comparing the certainty equivalent returns that suggest that, in order to change from a traditional portfolio to the one with commodities and currencies, an investor would likely request a higher premium. As the investor is getting more risk-averse, the interest in changing portfolios declines and the investor becomes more comfortable with the idea of keeping the existing traditional portfolio (Cotter, Eyiah-Donkor and Potì, 2017).

According to Cotter, Eyiah-Donkor and Potì (2017), the annual Sharpe ratio (excluding transactional costs) of a portfolio with added commodities is much lower than the Sharpe ratio of a traditional portfolio with a comparable strategy, therefore providing proof that there are no diversification benefits of adding commodities to a portfolio.

Cotter, Eyiah-Donkor and Potì's (2017) research shows conflicting results for in-sample and out-of-sample analyses. The mean-variance spanning tests conducted – while adding commodities, currencies or both to a traditional portfolio – show that diversification benefits do exist, although primarily close to the global minimum variance share of the frontier. However, this does not apply to the period of the commodity boom, since that was the period when commodities did not have diversification benefits. Higher Sharpe ratios are observed in portfolios including commodities, currencies or both, when taking return predictability into account. However, the out-of-sample analysis shows that there are no diversification benefits resulting from including currencies or commodities. According to Cotter, Eyiah-Donkor and Potì (2017), this might be due to the fact that in-sample analyses use flawless expected returns forecasts, that might exaggerate the diversification capabilities of those portfolios and might lead to false diversification results.

In the reseach by Abid, Dhaoui, Goutte and Guesmi (2019), conditional correlations between commodity and equity markets are observed and the best possible hedge ratio, based on different models, analysed. The main goal of this research is to show whether the projection of market volatility may be influenced by energy and precious metal commodities, bonds, the Fama and French HML (High minus Low) and SMB (Small minus Big) factors and the volatility index, which would in turn help hedge the risk exposure of investors and show that a better hedge of equity risk is linked to a portfolio based on Fama and French HML and SMB factors.

Abid, Dhaoui, Goutte and Guesmi (2019) highlight the three versions of the GARCH model – the dynamic conditional correlation model, the asymmetric dynamic conditional correlation model and the orthogonal model– all used to model the volatility and correlation dynamics, as well as hedge ratios between stock prices (S&P500) and oil, gold and bond prices, volatility index and the Fama-French SMB and HML factors. The research focuses on the time period from September 2006 to April 2017. The data used in Abid, Dhaoui, Goutte and Guesmi's (2019) research includes S&P 500 for stock prices, SMB for a return spread among small and big stocks, HML for a return of inexpensive stocks minus the return of expensive ones, oil and gold prices (with gold prices quantified in terms of futures contracts on gold of the Chicago Mercantile Exchange) and bond prices (quantified in terms of futures contracts on 10-year US treasury notes). The volatility index is used to measure the stock market volatility in the next 30 days on the market (Abid, Dhaoui, Goutte and Guesmi, 2019).

According to the conducted empirical research, oil has the highest variability (implied by the coefficient of variation), while gold has the lowest one. The volatility index has the highest standard deviation (Abid, Dhaoui, Goutte and Guesmi, 2019).

Significant positive correlation is observed between the equity market and the SMB and HML, while significant negative correlation is observed between equity and oil, gold, bonds and the volatility index. The greatest correlation is that between equities and the volatility index, implying that S&P 500 returns and volatility are negatively correlated. The correlations, apart from oil and bonds, show a decline from 2012 to 2015, followed by an increasing trend from 2015 onwards (Abid, Dhaoui, Goutte and Guesmi, 2019).

Due to positive dynamic conditional correlations between equities and SMB and HML throughout the observed period of time, it may be concluded that a company increasing in size or increasing capitalization lead to equity markets gaining money (Abid, Dhaoui, Goutte and Guesmi, 2019). Positive dynamic conditional correlations are observed between equities and oil. However, negative correlations were observed with the orthogonal model until September 2014, turning into positive correlations afterwards. It is outlined by Abid, Dhaoui, Goutte and Guesmi (2019) that the reason for those positive correlations could lie in the fact that the USA became one of the greatest oil and gas producers worldwide, with its increasing domestic oil production.

A negative dynamic conditional correlation is observed between equities and the volatility index in every GARCH model, indicating that increasing volatility might lead to equity markets losing money. Changing positive and negative dynamic conditional correlations are observed between equities and gold, as well as equities and bonds. The positive correlation between equities and gold can be due to various reasons, like the attraction to gold of developing and emerging countries; USA's higher demand for gold coming from China or India or the fact that gold is considered a superior inflation hedge and a *safe haven* asset. The reason for the negative correlation between equities and gold could be due to the fact that investing in gold is not considered the best of ideas in certain time periods without any turmoil, which leads to a decrease in gold investment in these periods, substituted with an increase in riskier investments (Abid, Dhaoui, Goutte and Guesmi, 2019). Regarding correlations between equities and bonds, the negative correlation between these two might be due to the lower return on 10-year treasury bonds.

The optimal hedge ratios between equities and oil, gold, bonds, volatility index, SMB and HML are analysed. The main hedging effects are observed for Fama and French HML and SMB factors, implying that the effects of company size and profitability-centered portfolios in hedging equity markets are substantial. The second best hedging effects are observed in gold and the third best in oil and bonds, implying that these assets offer good tail-risk hedge, due to the fact that they are considered *safe haven* properties. Lastly, the negative correlation effects between equity and the volatility index imply great economic benefits linked to ideal diversification (Abid, Dhaoui, Goutte and Guesmi, 2019).

The article by Abid, Dhaoui, Goutte and Guesmi (2019) analyses if oil, gold, bonds, Fama and French HML and SMB factors and the volatility index help predict the volatility of the equity market and hedge the risk exposure of investors. As a mean to improve the decision-making of investors, dynamic conditional correlation modelling between equities and other aforementioned assets is presented as a highly useful tool. The research also indicates that selective portfolios are a better option when hedging the investors' risk exposure. Additionally, hedging the investors' risk exposure can also be achieved by investing in gold, oil and bonds, owed to their *safe haven* nature.

According to this article's findings, the best model for analysing changing financial variables is the orthogonal model. Furthermore, strong diversification capabilities of commodities are also highlighted. The best hedge for the U.S. equity market is provided by gold, beating bonds and oil in the long run. These assets (oil, gold and bonds) depend considerably on equities over time, whether positively or negatively, highlighting their hedging benefits further. Additionally, it is concluded that the volatility index and equity markets have the opposite dynamics, implying massive diversification benefits. Hedge positions need to be reviewed and updated regularly, due to the changing dynamics between equity and bond markets, the oil and gold markets and the volatility index and their periodical dependencies (Abid, Dhaoui, Goutte and Guesmi's, 2019) . Nevertheless, it is additionally outlined by the authors that a good selection of company size and portfolio selection based on its profitability may lead to the best hedging benefits.

The article written by Fethke and Prokopczuk (2018) aims to analyse the diversification benefits of commodities added to a porfolio consisting of equities and bonds by conducting mean-variance spanning tests and out-of-sample portfolio optimizations. The research focuses on the question of whether an investor can improve the portfolio performance by adding commodity indices of the first, second and third generation to the portfolio. The first generation of commodity indices paints a passive image of the commodity market – and includes, for example, the S&P Goldman Sachs Commodity Index (S&P-GSCI), which is constructed in accordance to the world production amount of each underlying commodity. The second generation of commodity indices does not aim to represent the market and only includes long investments, as opposed to the third generation of commodity indices that accounts for short positions as well (Fethke and Prokopczuk, 2018).

The time period observed for the purpose of this research is from February 2000 to April 2017. Data used in Fethke and Prokopczuk's (2018) article includes commodity indices of the first (three), second (nine) and third generation (nine), based on total return estimations. Additionally, Fama and French SMB (Small minus Big) and HML (High minus Low) returns are used for equity market portfolios. 1-month U.S. treasury bills are used to represent the risk-free rate.

As per Fethke and Prokopczuk's (2018) research, first generation commodity indices are not considered good stand-alone investments, due to their low annual average return, high annual average volatility and a low Sharpe ratio. Higher average annual returns and lower average annual volatility, along with a higher Sharpe ratio, are noted with the second generation commodity indices. Higher downside risk and fatter tails when compared to normally distributed assets are observed in both generations of commodity indices, owed to their negative skewness and high kurtosis. Lastly, the third generation commodity indices have less volatility, with comparable returns to S&P 500 and a high Sharpe ratio. The positive skewness of the third generation commodity indices indicates more upside risk than is the case with prior generations (Fethke and Prokopczuk, 2018).

Additionally, the correlation between all observed indices and the benchmark assets was analysed as well. The findings imply that both first and second generation commodity indices show little to no correlation to traditional assets, making them a good addition to a portfolio containing stocks and bonds, as they would likely increase the diversification. The third generation of commodity indices has an even higher Sharpe ratio and more convenient correlations with other assets. In summary, the second and third generation of commodity indices show great diversification benefits, unlike the first generation, where an increase in performance is not clearly seen (Fethke and Prokopczuk, 2018).

According to Fethke and Prokopczuk (2018), diversification benefits of adding an asset to a portfolio are usually analysed via means of a spanning test. Therefore, spanning tests *Wald*, the *Lagrange Multiplier (LM)* and the *Likelihood Ratio (LR)* were used in this research for the case of two benchmark assets (S&P 500 and Barclays Bonds), by adding the 21 commodity indices to it – one at a time. The predefined null hypothesis of the article was that there is no statistically significant difference between the efficient frontiers of the original and the altered asset portfolio. The findings regarding the first generation commodity indices could not overturn the null hypothesis, excluding the weak risk reduction of the S&P-GSCI, implying that the first generation commodities are not enough to increase returns of a portfolio, but could nevertheless contribute to portfolio diversification, due to their low correlation properties. The findings regarding the second generation of commodity indices were similar, indicating that the second generation commodities are not successful in improving the efficient frontier. The findings regarding the third generation commodity indices implied a potential for portfolio improvement –however, mostly on the risk reduction side of things (Fethke and Prokopczuk, 2018).

This empirical research includes both an in-sample and an out-sample portfolio optimization analysis (Fethke and Prokopczuk, 2018). As a part of the in-sample analysis, two investor types were observed – an aggressive and a conservative investor type. The aggressive investor type allocated 24.55% to commodities, while the conservative investor type allocated 11.78% to commodities. After adding commodities to the in-sample portfolio, its performance improved (drastic improvement in Sharpe ratios) , regardless of the investor type. So, in conclusion, every generation of commodity indices can contribute to an improved in-sample portfolio performance.

On the other hand, as outlined by Fethke and Prokopczuk (2018), much more interesting are the findings of out-of-sample portfolio analyses, focusing on the impact of commodities on a portfolio consisting of bonds and equity over a certain time period. An out-of-sample portfolio could not be improved by adding first generation commodity indices to it. There were higher returns in the second generation of commodity indices in the out-of-sample portfolio, when compared to the benchmark and the first generation of commodity indices, in addition to a higher volatility of return. The results regarding the third generation of commodity indices were more heterogenous. In one half of the group, there were very high Sharpe ratios, even higher that those of first and second generation commodity indices. In the other half, returns were below the benchmark and other indices, in addition to a noticeable inability to reduce risk, leading to much lower Sharpe ratios than in the benchmark portfolios. The reason for such heterogenous results may lie in the allowing of short positions in the third generation of commodity indices (Fethke and Prokopczuk, 2018).

In Fethke and Prokopczuk's (2018) research, diversification benefits of 21 commodity indices were analysed. The findings of this research imply various diversification benefits of the first and second generation of commodity indices and even more significant diversification benefits of the third generation. The said findings were unaffected by the addition of HML and SMB portfolios to the benchmark assets.

The article's findings imply that the first generation commodity indices do not contribute to portfolio diversification in an equity-bond portfolio, regardless of their low correlation to other asset classes. Weak stand-alone investment capabilities and unimproved Sharpe ratios in an out-of-sample portfolio are linked to weak portfolio diversification of the first generation commodity indices. Second generation commodity indices performed better than the first generation – however, the increase in Sharpe ratio of the portfolio was not statistically significant. Third generation commodity indices are linked to unequal out-of-sample performances, as an inferior performance was noticed in one half of the sample in comparison with the benchmark. This might be due to short futures investments, which are conducted in this generation of commodity indices (Fethke and Prokopczuk, 2018).

In summary, the research shows that the second and third generation indices are better linked to higher diversification benefits than the first generation – however, there is variation to be seen in the rather comprehensive sample of these index groups (Fethke and Prokopczuk, 2018).

The article authored by Cheung and Miu (2010) aims to answer a few open questions regarding the diversification benefits of commodities – primarily, whether these benefits are (if at all present) significant. There is also the question of whether or not investors from the U.S. or Canada could benefit from investing in commodities. Additionally, are commodity futures affected by the change from bull to bear markets and are their diversification benefits long lasting (regardless of possible changes in the market)? Another question posed in the article is what kind of investor should seek to invest in commodities in general.

A part of Cheung and Miu's (2010) research includes the maximum Sharpe ratio of portfolios containing the three asset classes for both types of investors (a U.S. investor and a Canadian investor), calculated and then compared to the Sharpe ratio of the portfolio including commodity futures. A statistically significant increase of the Sharpe ratio implies that commodities do offer diversification benefits to a portfolio. This research does not consider short-selling (the weights vector is not negative), due to the fact that institutional investors usually do not tend to short-sell assets. Therefore, expected returns of commodity futures are modified in order to remain mean-variance efficient, even after adding the commodity futures (Cheung and Miu, 2010).

The data used in Cheung and Miu's (2010) research includes MSCI Canada, MSCI EAFE and MSCI US (representing equities) as well as 10-year U.S. government bonds (representing the government bonds). For commodity futures returns, an index created by Gorton and Rouwenhorst (2006) in their research paper "Facts and fantasies about commodity futures" is used in a study of Gorton and Rouwenhorst (2006, as cited by Cheung and Miu, 2010, p. 454). The time period in scope ranges from January 1970 to December 2005.

As implied by the research, commodity futures generally show a low correlation with equities and a negative correlation with bonds, leading to the conclusion that commodities provide good diversification benefits to an investor (Cheung and Miu, 2010).

As shown in the research, in both countries (USA and Canada), commodities lead to statistically significant (i.e. 5-percent level) diversification benefits when added to a portfolio. The statistical non-significance at the upper levels of risk-free rate imply that diversification benefits are in a decline for investors with higher risk acceptance (Cheung and Miu, 2010). The only difference between the two countries analysed is that adding commodities to a portfolio lead Canadian investors to invest more money into commodities instead of equities, due to a more resource-driven market in Canada, in which equities and commodities serve as substitutes to each other (Cheung and Miu, 2010).

The amount of diversification benefits is observed in different market regimes, as outlined by Cheung and Miu (2010). In order to observe diversification benefits of different market regimes, the commodity return data has to be divided in those two regimes. The low return state represents the bear and the high return state the bull commodity market.

As per the research, the low commodity return state is linked to low commodity volatility – and vice versa. Investing in commodities requires taking on more risk and is therefore compensated with higher returns, something that is not the case with international equities. The low states of commodity markets are linked with well-performing equity (US and EAFE) and U.S. bond markets, without significant volatility changes (Cheung and Miu, 2010).

As implied by the research, the correlation of different asset classes is slightly higher in bearish and slightly lower in bullish commodity settings. Due to the excellent commodity futures performance and low (and at times even negative) correlation with other assets throughout the high return state, an increase in risk-adjusted return is assumed when adding commodity futures to a portfolio (Cheung and Miu, 2010). However, it is clear that commodity futures offer significant diversification benefits, even though changes in the Sharpe ratio are not too significant statistically in cases of higher risk-free rates in the low return state. Portfolios containing higher risk-free rates are mainly selected by investors interested in higher risk-return portfolios (Cheung and Miu, 2010).

A similar analysis of these two states of commodity markets was undertaken with having Canadian investors in mind as well. Similar to results of their U.S. counterparts, Canadian equities and bonds show an increased performance without significant volatility changes in a low return state. However, due to the resource-driven nature of sectors in Canada, a minor difference in the mean return of equities was noted between the two states, unlike with the U.S. analysis (Cheung and Miu, 2010). Given the low performance of commodity futures and their high correlation with other assets in the low return state, it is clear that commodity futures do not offer significant diversification benefits in a low commodity return state (Cheung and Miu, 2010).

The analysis was repeated (without short selling) – however differing from the previous analysis in that it focused on changes in the regime of stock returns and not commodity futures returns. Two regimes were analysed, one with high volatility and low return states and another with low volatility and high return states. This analysis shows lower diversification benefits by commodity futures than in the first observation. With a risk-free rate of 0.5% or less, a higher Sharpe ratio was noted in the portfolio containing commodities in a high return state. On the other hand, commodity futures generally do not provide significant diversification benefits, higher than 5% in bearish markets, apart from the risk-free rate level of 0.1% (Cheung and Miu, 2010). When analysing Canadian investors, significant diversification benefits are noted in a bullish stock market, unlike in a bearish one.

Finally, the findings imply that, if the risk-free rate is high, this leads to insignificant diversification benefits of commodities, mainly in low commodity return states. High risk-free rates are mainly preferred by investors with a higher tolerance towards risk, which is why commodities are more popular among risk-averse investors (Cheung and Miu, 2010).

Cheung and Miu's (2010) research focuses on a few different issues. It concludes that commodities provide statistically significant diversification benefits in the long run. Additionally, it is highlighted that changes in commodity futures' behavior are regime-dependent (low return commodity futures environment linked to low volatility and vice cersa). In bearish stock markets, diversification benefits of commodities are very low, which leads to the conclusion that the real advantage of commodities lies in them increasing portfolio performance in infrequent outbreaks in the commodity market. Lastly, the findings show that diversification benefits are not generally applicable (for instance, the case of a resource-driven economy like that of Canada) and that investing in commodity futures is a better fit for risk-averse investors (Cheung and Miu, 2010).

In conclusion, the research by Cotter, Eyiah-Donkor and Potì (2017) focuses on whether a portfolio of stocks, bonds and treasury bills of a risk-averse investor can be improved in terms of risk and return by adding commodities and currencies. The research showed conflicting results for in-sample and out-of-sample analyses, where diversification benefits from including currencies or commodities in the in-sample analysis (excluding the commodity boom period) and no diversification benefits in the out-of-sample analysis exist. The empirical research by Abid, Dhaoui, Goutte and Guesmi (2019) indicates that hedging the investors' risk exposure can also be achieved by investing in gold, oil and bonds, owed to their *safe haven* nature. On the other hand, the empirical reseach by Fethke and Prokopczuk (2018) aims to analyse the diversification benefits of commodities added to a porfolio consisting of equities and bonds, by adding commodity indices of the first, second and third generation to the portfolio. Their research shows that the second and third generation indices are better linked to higher diversification benefits than the first generation. Lastly, the research by Cheung and Miu (2010) aims to answer questions regarding the diversification benefits of commodities and concludes that commodities provide statistically significant diversification benefits in the long run, that investing in commodity futures is a better fit for risk-averse investors and that changes in commodity futures' behavior are regime-dependent.

4.1.3. Risk reduction capabilities per commodity

The article authored by Belousova and Dorfleitner (2012) aims to analyse the diversification effects of numerous commodities on a portfolio of traditional assets for a Euro investor, arguing that diversification effects differ between different commodities – for instance, industrial metals, agricultural goods and livestock lead to risk reduction, whereas commodities like energy and precious metals lead not only to risk reduction but also improve returns. Generally, this article concludes that investors can profit from diversification benefits by investing in commodities, whether via means of financial instruments or physical commodity and commodity futures.

Since diversification benefits of commodities can be influenced by changes in the exchange rate, research by Belousova and Dorfleitner (2012) focuses specifically on a Euro investor, unlike much of prior research in this field. Additionally, the research differs from previous ones in that it analyses 25 individual commodities instead of analysing commodity indices, in order to get a more objective view on individual commodities and commodity sectors. The time scope of the research is a period of 17 years, covering both the periods of rising as well as the periods of falling equity markets.

Data used in this empirical research is for the period from January 1995 to December 2010, with all denominations in USD converted into EUR. For the purpose of this research, the MSCI Europe is chosen to represent the European equity market, the MSCI US for the U.S. equity market, the MSCI Pacific for the Asia-Pacific market and the MSCI EM for the emerging markets. The European debt Market is represented by the J.P. Morgan EMU government bond index. The risk-free rate is approximated with EURIBOR (Belousova and Dorfleitner, 2012).

The commodities used in this research are chosen according to the composition of the Dow Jones-UBS Commodity Index (DJ-UBSCI; index that is sufficiently diversified due to its obligatory weight limits) and the S&P Goldman Sachs Commodity Index (S&P GSCI; index that is mostly focused on the energy sector with a weight of 70%), as noted by Belousova and Dorfleitner (2012).

The research analyses 25 individual commodities, making sure that both abovementioned indices are represented (with 24 components of the S&P GSCI index and 18 of the DJ-UBSC). Additionally, platinum is added to the sample, in order to strengthen the representation of the precious metals sector. The commodity sample used ranges from the energy sector to softs and grains, industrial metals, precious metals and the livestock sector (Belousova and Dorfleitner, 2012).

The research conducted by Belousova and Dorfleitner (2012) indicates that almost all commodities have positive excess kurtosis, which implies a higher probability for extreme events, due to fatter tails of their returns. Most of commodities are positively skewed, which indicates lower downside risk and the rising return bias of a portfolio, unlike with stocks and bonds.

As part of the article written by Belousova and Dorfleitner (2012), correlation coefficients of individual commodities and benchmark assets are analysed. The findings suggest that a significant reduction in portfolio volatility may be achieved if commodities are introduced into a portfolio consisting of stocks and bonds. This is due to the fact that almost all commodities are negatively correlated with fixed income securities and money markets, while at the same time being negatively or mildly positively correlated with equity markets (Belousova and Dorfleitner, 2012). The reason for this low correlation between commodities and other traditional assets could be that commodities are a separate asset class and are therefore affected by other economic fundamentals than other assets, additionally strengthening the low correlation between those asset classes. Generally, the research argues that commodities are volatile assets, with lower Sharpe ratios, that are a very valuable addition to the portfolio because of their diversification potential, owed to their negative or slightly positive correlation with other assets (Belousova and Dorfleitner, 2012).

In this article, the diversification potential of commodities is analysed by using the so-called *spanning tests*. These tests indicate that commodities have a significant influence on the diversification of portfolios with traditional assets. All commodities analysed in the energy sector, with the exception of natural gas, show a decrease in risk and an increase in portfolio return. Significant diversification properties are noticed in precious metals as well. The highest Sharpe ratio was measured in the energy and precious metals sector, resulting in a major contribution to the improvement of the portfolio return (Belousova and Dorfleitner, 2012).

As stated by Belousova and Dorfleitner (2012), diversification properties in other commodity sectors are only linked to a decrease in risk levels. In the industrial metal sector, this can be seen with aluminium and copper, due to their lower correlation levels with traditional assets. Diversification properties are seen in the softs sector (cocoa and sugar), grains sector (corn, soybeans and Kansas wheat all lead to a reduction in portfolio volatility) and livestock sector (steers and feeder cattle). Especially strong risk reducing capabilities are found in white sugar, live steers and feeder cattle.

In summary, diversification properties of commodities depend on the sector in question. However, it should be noted that the energy sector and the precious metal sector influences the portfolio performance positively when it comes to both risk and return. The benefits of including agricultural commodities and livestock in a portfolio are mainly connected to risk reduction and are therefore a more interesting option for risk-averse investors. Strong diversification abilities are seen in the majority of commodities in the sample. Just 6 out of 25 (cotton, Chicago wheat, lead, nickel, zinc and pork bellies) do not show any diversification properties, indicating that an investment in commodities is positively influencing the portfolio performance of a traditional portfolio (Belousova and Dorfleitner, 2012).

One of the main goals of Belousova and Dorfleitner's (2012) research is to analyse the diversification impact of commodity futures by conducting spanning tests with twenty-five S&P GSCI excess return sub-indices on certain commodities. The results of these spanning tests overlap with those of tests undertaken on physical commodities, implying that adding commodity futures to an investment portfolio might yield the same benefits as adding physical commodities to the portfolio (Belousova and Dorfleitner, 2012). Commodities from the energy and precious metal sectors contribute positively to a portfolio performance in terms of both risk and return, whereas commodities from other sectors are used more for the hedging of the portfolio risk and are therefore better suited for risk-averse investors (Belousova and Dorfleitner, 2012).

The time period in scope for Belousova and Dorfleitner's (2012) research is divided in two different time slots: that of increasing and that of decreasing equity markets (bull and bear markets), in order to investigate the diversification abilities of commodities under two sets of entirely different conditions. It is apparent that, in bull markets, a contribution to the return of an investment portfolio is made by adding crude oil, gas oil and heating oil to the portfolio, with heating oil also contributing to risk reduction over the same time period. On the other hand, in bear markets, the contribution to diversification benefits of energy commodities arises from risk reduction. In bull markets, precious metals contribute to lower portfolio volatility. Silver additionally influences the portfolio return positively. However, in bear markets, platinum fails to keep its diversification capabilities, while gold and silver contribute to risk reduction. With regard to the industrial metal sector, only aluminium has diversification capabilities in bull markets. Stronger diversification capabilities of industrial metals are observed in bear markets. In bear markets, the presence of soft commodities does not lead to any investment benefits, whereas their presence in bull markets leads to strong diversification capabilities. The commodities of the softs sector (cocoa and sugar), livestock sector (feeder cattle and live steers) and all commodities in the grains sector add to risk reduction in a portfolio. Hence, the diversification capabilities of the grains, softs and livestock sectors are linked to times of rising markets (Belousova and Dorfleitner, 2012). Since differences in the diversification benefits exist, a tactical allocation strategy concerning the price movements in equity markets would boost the diversification benefits of a portfolio.

Belousova and Dorfleitner (2012) argue that the strongest indication of diversification may be seen in the energy sector and the precious metal sector. When talking about the energy sector (crude oil, gas oil, heating oil and unleaded gasoline), similar diversification benefits are noticed, including a simultaneous decrease in the risk and increase in the portfolio return. Diversification benefits of the natural gas could not be confirmed. In the precious metal sector, diversification benefits are high, with the notable exception of platinum (somewhat weaker diversification capabilities than gold and silver), especially in bear markets. Diversification capabilities of aluminium are remarkable when it comes to the industrial metal sector. In the grains sector, no significant difference is to be seen between separate commodities, as they all have an impact on risk reduction in a portfolio. Regarding the softs sector, cocoa and sugar show diversification benefits, while, in the livestock sector, feeder cattle and live steers have a positive impact on portfolio performance, unlike pork bellies that do not add to the diversification capabilities of a portfolio (Belousova and Dorfleitner, 2012).

In summary, Belousova and Dorfleitner's (2012) research analyses 25 commodities with regard to their diversification capabilities, resulting in no definitive overall conclusion. Risk reduction in a portfolio could be linked to the addition of agricultural, livestock and industrial metal commodities to a portfolio, especially attractive to risk-adverse investors, with a more prominent allocation on the conservative part of the efficient frontier, with portfolios near the global minimum variance portfolio (GMVP). However, diversification benefits can be improved by adjusting the management strategy in accordance with price movements on equity markets (Belousova and Dorfleitner, 2012).

In bear markets, weak diversification capabilities are noticed in softs, grains and livestock sectors, whereas industrial metals have almost no diversification capabilities in bull markets. The most valuable addition to a portfolio are commodities coming from the energy and precious metal sectors, since they are beneficial for the portfolio in both bear and bull markets in terms of both risk and return, making them a good choice for any investor (Belousova and Dorfleitner, 2012).

Furthermore, in the research paper written by Heidorn and Demidova-Menzel (2011), the impact of adding gold to a traditional portfolio – especially in the light of the increase of gold prices from the year 2000 onwards – is observed. An important benchmark when it comes to the fluctuation of gold prices happened in the year 2004, when European central banks agreed to limit the selling of gold in the years to come, with the outcome turning out to be a lower supply and – hence – a higher price.

As argued by Heidorn and Demidova-Menzel (2011), gold as a commodity contributes to the diversification of a portfolio, as its price moves in an entirely different manner when compared to prices of other asset groups, like stocks or bonds, contributing significantly to the general risk-return characteristics of the portfolio in question. On the other hand, gold also had quite negative returns for a long period of time – until 2002, when this negative trend changed. The time period between 1993 and 1999 serves as an exception to this rule, being that these few years were a relatively good time period for gold (Heidorn and Demidova-Menzel, 2011).

According to Heidorn and Demidova-Menzel (2011), the time period from 1991 to 2006 in the Eurozone was marked by an especially low correlation of gold with the REXP (the German government bond index) and the DAX (the German share index), leading to a conclusion that gold contributes to a better diversified portfolio.

In the research, an efficient portfolio with the ideal risk-return rate in the Eurozone, as according to Markowitz's modern portfolio theory (MPT), is presented. The top left part of the return-standard deviation graphic is considered the most attractive part, due to its higher return and lower risk characteristics. The addition of gold to an investment portfolio over different periods of time is analysed in this research as well (Heidorn and Demidova-Menzel, 2011).

As per their research the contribution of gold to an overall risk adjustment over the entire time period observed (from 1988 to 2006) is rather weak. But even if choosing to disregard this low correlation of gold, its low return of below 2%, combined with a very high volatility, makes it highly unattractive as a possible addition to the investment portfolio. An investment portfolio consisting of just bonds and equity would be a far more attractive option over this specific time period (Heidorn and Demidova-Menzel, 2011).

In the time period from 1990 to 1999, gold had negative returns, implying that investors would actually decrease their investment performance and increase the risk if opting for the inclusion of gold in the portfolio, making gold a very unattractive investment alternative over this time span. However, unlike gold, equity returns were very high in the 1990s and equity-bond investment portfolios therefore a much more attractive alternative (Heidorn and Demidova-Menzel, 2011).

From the year 2000 onwards, there was a noticeable shift in the efficient line to the left, owed to an overperformance of gold (more than 7% return with low volatility), making gold a very attractive investment option in this time period. In contrast, returns on equity markets were rather low over the same time period (Heidorn and Demidova-Menzel, 2011).

A similar analysis was conducted for the United States as well. Compared to the data from the Eurozone, the returns of gold were much higher than the returns in EUR, accompanied by high volatility. The returns of gold were high in the time period from 1974 to 1979 and then again from 2000 to 2006, and negative gold returns have marked the time period from 1980 to 1989 and then again from 1990 to 1999, as outlined by Heidorn and Demidova-Menzel (2011). The analysis of this research shows that, up until 2006, a portfolio mix of bonds and equities was the most beneficial one for an investor and that over the 1990s, the volatility of gold had plummeted – however, due to its negative returns, gold was not yet considered an attractive investment option. A movement in the efficiency line to the left, owed to the addition of gold to the portfolio, took place in the time period from 2000 to 2006, due to high returns on gold (over 12%) and very low returns on equity, making gold an attractive portfolio investment opportunity as outlined by Heidorn and Demidova-Menzel (2011).

In summary, after European central banks decided to limit the selling of gold in the year 2004, the supply dropped, leading to an increase in the price of gold – and an overall decrease in dependency of supply and demand from central banks. Even though the price of gold started to increase, the correlation remained low, in addition to a similar level of volatility as with equity, which made gold an attractive investment opportunity in the time period from the year 2000 onward (Heidorn and Demidova-Menzel, 2011).

Furthermore, the research authored by Bredin, Conlon and Potì (2017) focuses on the decreasing of the downside risk via means of combining precious metals – such as gold, silver and platinum – with equities, in addition to exploring their influence on portfolio risk-adjusted return. It has been discovered that commodities add to the short-term downside risk reduction, while simultaneously contributing to the rise of the long-term portfolio risk. It should be noted that investing in futures of precious metals may be considered as good a diversification option as investing into physical metals, being that their variance and kurtosis characteristics contribute to downside risk reduction, as outlined by Bredin, Conlon and Potì (2017).

This article examines the capability of gold, silver and platinum to decrease portfolio downside risk when combined with equities, in addition to exploring what happens to the price of portfolio diversification by adding precious metals, in terms of change in risk and return of an equity portfolio with precious metals as compared to an equity portfolio (Bredin, Conlon and Potì, 2017).

The article by Bredin, Conlon and Potì (2017) argues that the impact of gold on risk reduction within a portfolio is higher than previously thought, but only when it comes to short-term investments (of up to 15 days). The impact of silver and platinum did not prove to be as considerable as that of gold and is even weaker when observed in the long run. This stresses the importance of choosing the right precious metal to invest in when looking to make a long-term investment. The conclusion of the article by Bredin, Conlon and Potì (2017) is that the investor has to pay to accomplish downside risk reduction, as opposed to earlier research according to which investors should earn a risk premium for enduring downside risk.

Platinum bullion spot prices, used in this empirical research by Bredin, Conlon and Potì (2017), come from the London Platinum Free Market, while the gold and silver prices come from the London Bullion Market Association (LBMA). The prices for gold and silver futures are provided by Exchange, Inc. (COMEX) and the platinum futures prices by the New York Mercantile Exchange (NYMEX). The ETFs used for the analysis include those where the provider holds an underlying physical precious metal, i.e. the SPDR gold exchange traded fund (in the period from 2004 to 2014) and the Ishares Silver Trust ETF (in the period from 2006 to 2014), as outlined by Bredin, Conlon and Potì (2017). Standard and Poor's 500 (S&P 500) total return index for the period from 1980 to 2014 is chosen to represent the equity prices and the U.S. one-month Treasury Bill for the risk-free rate. A correction of the measurement was necessary in order to take factors such as seasonal effects into account. This was done by calculating a cross-sectional average across intervals of length. For example, the weekly standard deviation is calculated by averaging the standard deviations that were calculated five times (with each one corresponding to a weekly estimated risk for each day), as stated by Bredin, Conlon and Potì (2017).

According to Bredin, Conlon and Potì (2017), the article examines the capability of precious metals to reduce portfolio downside risk when invested in combination with S&P 500, with a focus on the following:

- Risk reduction for a portfolio consisting of 10% precious metals and 90% equities;
- Impact of diverse allocation weights of precious metals on risk reduction;
- Whether changes in results occur over different periods of time;
- Whether risk reduction capabilities are linked to different periods of time.

A decrease in risk at different confidence intervals throughout a certain period of time is noted, with the assumption being that a total of 10% of the portfolio is allocated to gold, silver and platinum and the other 90% to S&P 500. The decrease in risk in this instance is measured by comparing such a scenario to having a portfolio made up exclusively of S&P 500. Slight variations in the level of volatility are identified throughout the examined time period (Bredin, Conlon and Potì, 2017).

The research indicates that a maximum risk reduction for the greatest risks, i.e. the ones with the confidence level of 99% and 99.9%, occurs in the short term. The decrease in risk does not remain constant in the long run, particularly when it comes to silver and platinum. Therefore, the article argues that a chance of risk reduction via means of precious metals could very well exist in the short run but could also have downside effects in the long run – at least when investing in platinum and silver (Bredin, Conlon and Potì, 2017).

The impact of varying allocation weights (between 1% and 30%) of precious metals on risk reduction at a 99% confidence level is examined (Bredin, Conlon and Potì, 2017). The research implies that only gold offers a continuous, yet non-linear decrease in risk for greater allocations. In case of silver and platinum, the influence of greater allocations differs between short-term and long-term investments.

Whether there are changes in risk reduction over different time periods is one of the points covered in the article as well. The research indicates that the level of VaR risk reduction for periods longer than 10 days was somewhat weaker from 2003 to 2014. As for silver and platinum, the portfolio downside risk increases in the long run, unlike gold that shows a downside risk reduction over all intervals, albeit with a weakening tendency (Bredin, Conlon and Potì, 2017).

Dynamic capabilities of precious metals to reduce risk are analysed in this research as well. The calculation of the risk reduction was conducted per day, using a window of 1.250 days, for both the one-day and the 20-day interval, as well as for allocation of 10% and 20% of gold, silver and platinum (Bredin, Conlon and Potì, 2017). The research indicates that a higher decrease in risk is provided by precious metals in the short run. A less steady risk reduction, even resulting in an increase in risk on a few occasions, is noticeable in case of silver and platinum. Higher allocations to precious metals are found to result in risk decrease on most occasions, however at the cost of increased volatility in relation to risk decrease (Bredin, Conlon and Potì, 2017).

The price of downside risk reduction was studied by Bredin, Conlon and Potì (2017) as well, as they have tested the influence on risk-adjusted returns in case of 10% of the portfolio being allocated to precious metals. Adding gold to the portfolio results in the decrease of the Sharpe ratio of between 9% and 12%, with slight differences within the observed timeframe. A 15% reduction in downside risk was observed in the 1-day-interval – however, at the price of losing 9% of risk-adjusted returns. In the 60-day-interval, giving up on 11% of risk-adjusted returns would reduce the downside risk by 9%. These results additionally support the notion that gold serves as a good short interval hedge against downside risk. In case of silver, in the one-day interval, losing 23% of risk-adjusted returns would reduce 20% of downside risk. In the 60-day interval, a decrease of 1% of downside risk means a significantly higher decrease of risk-adjusted returns. For platinum, the influence on risk-adjusted returns in the 1-day interval is marginal – however, the investor would be able to decrease the downside risk to 86% of an equity-only portfolio (Bredin, Conlon and Potì, 2017).

A relative modified Sharpe ratio and a relative VaR with a 99% confidence level for different weights of precious metal allocations (from 1% to 30%) is analysed as well. It is assumed that the remaining part of the portfolio is invested in S&P 500. In case of gold, as the weight increases to 30%, the relative VaR decreases to 0.54 times of an equity-only portfolio and the risk-adjusted returns to 11%. Looking at a 30-day horizon, the portfolio risk decreases to 0.785 times of an equity-only portfolio, however with a price of surrendering 47% of risk adjusted returns – once again in favor of the theory that diversification benefits of gold are mostly short-term-related. In case of silver and platinum, the decrease in risk found is highest in the short run, suggesting that gold is the sole precious metal that may be used in decreasing downside risk over all intervals and that risk averse investors are even willing to pay more in order to decrease the risk by holding precious metals (Bredin, Conlon and Potì, 2017).

The article by Bredin, Conlon and Potì (2017) examines how an allocation to precious metals impacts the moments of a portfolio's distribution and their influence on extreme downside risk. These risk reduction capabilities of precious metals were analysed in the context of a two-moment VaR (accounting for mean and standard deviation), a three-moment VaR (additionally accounting for skewness) and a four-moment VaR (further accounting for the impact of kurtosis in downside risk reduction) with the assumption of a 10% allocation to precious metals and a 99% VaR confidence level, as outlined by Bredin, Conlon and Potì (2017). This analysis indicates that gold has the highest risk-decreasing properties of all precious metals examined, as it provided a consistent 2-moment VaR decrease over all examined intervals.

As emphasised by Bredin, Conlon and Potì (2017), both futures and ETFs were more frequently used by investors of precious metals in recent times, as investors try to avoid costs of storage and security connected to investment in physical precious metals – even though the risk reduction properties of those have not been examined in depth as of yet. According to this article, the possible decrease in variance through using futures and GSCI (Goldman Sachs Commodity Indices) is similar to using physical metals, however an investor could profit from 1% to 2% in additional decrease of downside risk if investing in physical metals. On the other hand, when investing in futures or GSCI, an investor does not have to accept the same decrease in risk-adjusted returns as an investor who invests in physical metals.

In summary, the article by Bredin, Conlon and Potì (2017) raises the following points:

- In the medium and short term, investing in gold leads to a strong downside risk decrease. On the other hand, silver and platinum show strong risk reduction characteristics only in the short term. In the long term, silver and platinum may even lead to a higher downside risk for higher allocations.
- Non-significant risk reduction contributions from precious metals variance over all observed intervals.
- Decreased portfolio risk in the short term is owed to kurtosis properties of precious metals. On the other hand, an increase in portfolio risk in the long run was noted as well.
- Decreased Sharpe ratio and modified Sharpe ratio of a portfolio including precious metals compared to an equity-only portfolio, in the period from 1980 to 2014, was observed as well. This research suggests that investors are willing to give up on returns in order to decrease the probability of high-tail losses, contradicting earlier research in the field.

Investing in ETFs and futures of precious metals is considered another good diversification option, along with investing in physical metals.

To summarize, when it comes to risk reduction capabilities per commodity, the research by Belousova and Dorfleitner (2012) gives a good overview of the topic, by arguing that diversification effects differ between different commodities and showing that including industrial metals, agricultural goods and livestock leads to risk reduction, whereas commodities like energy and precious metals lead not only to risk reduction but to improved returns, making them a good choice for any investor. The research paper written by Heidorn and Demidova-Menzel (2011) analyses the impact of adding gold to a traditional portfolio and concludes that, after European central banks decided to limit the selling of gold in 2004, the supply dropped, leading to an increase in the price of gold while the correlation remained low, in addition to a similar level of volatility as with equity, which made gold an attractive investment opportunity in the time period from the year 2000 onward. Lastly, decreasing of the downside risk via means of combining precious metals – such as gold, silver and platinum – with equities, in addition to exploring their influence on portfolio risk-adjusted return is addressed in the research authored by Bredin, Conlon and Potì (2017). The research concludes that there are risk reduction capabilities and that they differ per commoditiy and investment horizon, i.e. in the medium and short term, investing in gold leads to a strong downside risk decrease, silver and platinum show strong risk reduction characteristics only in the short term while in the long term, silver and platinum may even lead to a higher downside risk for higher allocations.

4.2. Advantages and disadvantages of investing in commodities

According to Chambers, Black and Lacey (2018), the main reasons why an investor would choose to invest in commodities include protection from inflation, diversification benefits and potentially higher returns. Being that commodity prices are largely influenced by prices in the energy sector, something that impacts wholesale and consumer prices as well, commodities are considered to at least provide a modest inflation protection (Chambers, Black and Lacey, 2018). As per Kayser, Paris and Ross (2011), commodities are traditionally linked to very low correlations to equities and combinations of equities and bonds. On the other hand, as per Chambers, Black and Lacey (2018), there are known to be positive correlations between prices of physical commodities and the overall economic activity. There are two different drivers behind the motivation to improve the return by investing in commodities: (1) expecting a higher return for the non-traditional risk taken or (2) expecting a higher return based on the belief that commodities are worth more than what was paid for them initially (Chambers, Black and Lacey, 2018).

Kayser, Paris and Ross (2011) acknowledge another additional aspect that should be taken into account when deciding whether or not to invest in commodities – and that is the fact that prices od physical commodities are influenced by their own supply and demand, in addition to a myriad of other different local and global factors (including the political situation, environment, weather, labor problems, currency and technological changes) possibly impacting the demand for commodities.

As per carefully selected empirical research highlighted over the course of this chapter, there are many different advantages and disadvatages when it comes to investing in commodities. According to Heidorn and Demidova-Menzel (2011), gold is an attractive investment opportunity from the year 2000 onwards, due to its low correlation to equity. Belousova and

Dorfleitner (2012) claim that no general conclusion can be drawn for all commodities analysed, as there are noticeable differences between them. Risk reduction in a portfolio may be linked to adding agricultural, livestock and industrial metals commodities to one, which is particularly applicable to risk-adverse investors, that allocate more on the conservative part of the efficient frontier and hold portfolios near the GMVP. Weak diversification capabilities in softs, grains and livestock sectors are seen in bear markets, whereas industrial metals have almost no diversification capabilities in bull markets. The most valuable addition to a portfolio are commodities from the energy and precious metals sectors, as they are beneficial for the portfolio in both bear and bull markets in terms of both risk and return, making them a good choice for both conservative and aggressive investors.

However, different risk reduction capabilities are seen in different precious metals over different time intervals. According to Bredin, Conlon and Potì (2017), in the medium and short term, investing in gold leads to a strong downside risk decrease. On the other hand, silver and platinum show strong risk reduction characteristics only in the short term. In the long term, silver and platinum may even lead to a higher downside risk for higher allocations. Investors are willing to give up on returns in order to decrease the probability of high tail-losses, contradicting much of the earlier research in the field (Bredin, Conlon and Potì, 2017).

Öztek and Öcal's (2017) research further highlights the reasons for investing in commodities by arguing that high gains arise from a portfolio diversification between commodity and stock markets, as opposed to investing only in the stock market. Also, this way, according to the article in question, a portfolio provides better improvements during quieter rather than in more volatile periods. Lastly, the research suggests that markets are not integrated enough to have a lasting trend when it comes to correlation and that they are rather dependent on market volatility.

The empirical research conducted by Narayan,Thuraisamy and Wagner (2017) finds that there is a delayed cross-market pricing transmission from gold to bonds – and consequently to oil and inflation as well – suggesting that macroeconomic uncertainty influences the equity market, then the bond market and then comes back around to the measure of uncertainty. Therefore, these authors claim that there is an interaction between bond, equity and commodity cycles.

The diversification benefits of commodities are analysed in Cotter, Eyiah-Donkor and Potì's (2017) article as well, with differing results for the in-sample and the out-of-sample analysis. The in-sample analysis shows that the addition of commodities, currencies or both to a traditional portfolio yields diversification benefits – however, mainly close to the global minimum variance share of the frontier. This does not apply to periods of commodity booms, since commodities do not have diversification benefits in these periods. Higher Sharpe ratios are seen in portfolios including commodities, currencies or both, after accounting for return predictability. However, the out-of-sample analysis shows no diversification benefits from including currencies or commodities in a portfolio. According to Cotter, Eyiah-Donkor and Potì (2017), this may be due to the fact that in-sample analyses use flawless expected returns forecasts, that tend to exaggerate diversification capabilities of such portfolios and may in turn lead to false diversification results.

Diversification benefits of investing in commodities are not generally applicable and are either dependent on the regime or the investment period, as per Cheung and Miu's (2010) research. It is concluded that commodities provide statistically significant diversification benefits in the long run. Additonally, it is noted that changes in the commodities futures behavior are regime-dependent (low return commodity futures environment linked to low volatility and vice cersa). In bearish stock markets, the diversification benefits of commodities are very low, which leads to the conclusion that the real advantage of commodities in infrequent outbreaks of the commodity market is the increase in portfolio performance (Cheung and Miu, 2010).

An analysis of Main, Irwin, Sanders and Smith (2018) on whether the risk premium of long-term commodity investors is affected by the financialization of commodity futures markets resulted in the conclusion that returns in commodity futures markets are mainly driven by individual random supply and demand fluctuations, since the average level of unconditional risk premiums mainly is not influenced by the financialization of the commodity futures market in the mid-2000s.

Therefore, when discussing the advantages and disadvantages of commodity investing and their hedge benefits, no universal answer can be offered. As per Abid, Dhaoui, Goutte and Guesmi (2019) hedge positions should be reviewed and updated regularly, due to the changing dynamics between the equity and bond markets, the oil and gold markets and the volatility index and their periodical dependencies. Nevertheless, it is additionally outlined that a good selection of company size and portfolio selection based on its profitability might bring about the best hedging benefits (Abid, Dhaoui, Goutte and Guesmi, 2019). Diversification benefits of commodities are different with different generations of indices, as noted in Fethke and Prokopczuk's (2018) research that included commodity indices of the first, second and third generation. The conclusion is that the second and third generation indices are better linked to higher diversification benefits than the first generation – however, the comprehensive sample indicates variation within the group of indices chosen (Fethke and Prokopczuk, 2018).

5. Empirical analysis

This chapter aims to provide a detailed outline of the empirical analysis conducted – and consists of the methodology and data used, three main parts of the seventeen-year empirical analysis (the risk and return analysis for the used indices, the data normality tests, the portfolio optimization calculated for three different scenarios) as well as of an additional five-year empirical analysis and a conclusion on whether or not adding commodities to a portfolio enhances its quality in terms of return and volatility.

5.1. Methodology and data

According to Bank Investment Consultant (2006), the shortest period to be analysed in order to get a material statistical mean return is three years, although a five-year timeframe would likely be more adequate for this purpose. Having this in mind, a significantly longer time period of seventeen years (from 02.01.2004 to 01.01.2021) was chosen for the primary empirical analysis of this thesis. The data used in this empirical analysis, i.e. the index prices of the abovementioned four indices, were obtained through Refinitiv (2021).

Quantitative research methods were used for the empirical analysis presented below. The prices of four total return indices, all denominated in the same currency (USD), each representing the various markets (commodity, bond and stock market) were chosen. Quantitative analysis of this data was performed partly by Microsoft Excel and RStudio. In order to calculate the risk and return of potential investment portfolios subject to this empirical analysis, weekly prices from 02.01.2004 to 01.01.2021 were used. This data was used in all further risk and return analyses, data normality tests and portfolio optimization calculations. Weekly index prices from 01.01.2016 to 01.01.2021 of the same four total return indices used before were used in the additional five-year empirical analysis.

The following indices were selected for this empirical analysis, each of them representing a specific asset class and market (stocks, bonds and commodities):

a) MSCI World Index - Total Return for stock price movements;
b) FTSE World Government Bond Index - Total Return for bond price movements;
c) S&P GSCI Commodity Index - Total Return for commodity price movements;
d) Rogers International Commodity Index - Total Return (RICI) Index for commodity price movements.

The reason why two different commodity indices were used in this calculation is because they both offer a uniqe set of characteristics and using both in calculations offers a possibility to compare the effects that different commodity indices may have on portfolio optimization. On one hand, the Rogers International Commodity Index (RICI) is a highly stable index, with broad and consistent components, as argued by Beeland Interests (2021) while, on the other hand, the S&P GSCI Commodity Index has a very high share invested in the energy sector, making it co-dependent with the movements on the energy market, as argued by by Boerse. de (2021).

The empirical analysis consist of four main parts: the risk and return analysis, data normality tests, the portfolio optimization and the five-year empirical analysis. In the first part (risk and return analysis), the abovementioned four indices were compared in the period from the beginning of 2004 until the end of the year 2020 (seventeen years in total), in terms of their return and volatility. Additionally, the correlation and covariance between these three asset classes were calculated. In the second part of the empirical analysis, various data normality tests were conducted, in order to test whether data normality can be assumed for the given data set or not. These normality tests entailed a skewness and kurtosis analysis, the Kolmogorov-Smirnov test and the Shapiro-Wilk test, as well as a histogram and a Q-Q Plot of the data used in the empirical analysis.

In the third part, different portfolio optimization calculations (three different scenarios) were performed and their results analysed:

Scenario 1

In the first scenario, the portfolio is composed exclusively of stocks and bonds.

Scenario 2

In the second scenario, the S&P GSCI Commodity Index - Total Return is added to the portfolio from *Scenario 1*, so that the portfolio now consists of stocks, bonds and commodities.

Scenario 3

In the third scenario, the Rogers International Commodity Index - Total Return (RICI) Index is added to the portfolio from *Scenario 1*, so that the portfolio now consists of stocks, bonds and commodities.

For all scenarios, Sharpe ratios and Value at Risk (VaR) calculations were performed and results of all scenarios analysed and compared in terms of their risk and return profiles.

The timeframe of the empirical analysis is set from 02.01.2004 until 01.01.2021, owing to the following reasons:

- The year 2004 is chosen as the starting year in this timeframe because that is when the European central banks agreed to limit the gold sales, leading to a lower supply and higher prices of gold, as outlined by Heidorn and Demidova-Menzel (2011) and further explained in Sub-chapter 4.1.3);
- The year 2004 is also the year when the financialization of commodity markets started, as outlined by Main, Irwin, Sanders and Smith (2018);

- The data timeframe includes the commodity 'boom' period (from 2000 to 2014), during which commodities did not provide diversification benefits, according to Cotter, Eyiah-Donkor and Potì (2017), see Sub-chapter 4.1.2;
- Unusual one-off events, like the financial crises of 2008 and 2011 (see Sub-chapter 3.2), as well as the latest anomalies caused by the COVID-19 pandemic were included (completely or – as in the case of the COVID-19 pandemic – only partially, being that the observed timeframe ends with the year 2020) in the timeframe of this empirical analysis.

In the fourth part of this empirical analysis, an additional five-year empirical analysis was conducted, with a focus on a risk and return analysis (comparing the abovementioned four indices in terms of their return and volatility in the period from 01.01.2016 to 01.01.2021), a correlation and covariance analysis and different portfolio optimization calculations (three different scenarios), by using data from a shorter (five-year) time span.

5.2. Indices used in the empirical analysis

Weekly prices of the following indices were used for this empirical analysis, each of them representing a specific asset class and market (stocks, bonds and commodities), in the period from 02.01.2004 until 01.01.2021:

a) MSCI World Index - Total Return for stock price movements;
b) FTSE World Government Bond Index - Total Return for bond price movements;
c) S&P GSCI Commodity Index - Total Return for commodity price movements;
d) Rogers International Commodity Index - Total Return (RICI) Index for commodity price movements.

A summary of the main information about the two commodity indices used – the Rogers International Commodity Index - Total Return (RICI) and the S&P GSCI Commodity Index - Total Return – is presented in Sub-chapters 2.4.3 and 2.4.2, respectively. The other two indices used in this empirical analysis – the MSCI World Index - Total Return and the FTSE World Government Bond Index - Total Return – are described below.

5.2.1. MSCI World Index

The MSCI World Index is one of the most important indices worldwide for developed countries and one of the easiest paths to a broadly diversified equity investment (Handelsblatt, 2021). The index includes large and mid-sized companies over twenty-three developed countries worldwide, covering the majority of the free float-adjusted market capitalization in those countries (MSCI World Index, 2021). The highest country weight in the MSCI World Index is assigned to the USA, with 66.57%, followed by Japan with 6.9% and the United Kingdom with 4.38%, whereby the highest sector weight is assigned to information technology with 20.98%, followed by financials with 14.23% and health care with 12.37%, as outlined in the MSCI World Index (2021).

5.2.2. FTSE World Government Bond Index

The FTSE World Government Bond Index is the leading fixed income index of the FTSE created in the year 1986 and it is a commonly used benchmark index, as outlined by Refinitiv (2021). It captures the performance of sovereign bonds that are investment grade bonds with a fixed rate in a local currency (Refinitiv, 2021). The FTSE World Government Bond Index captures sovereign debt from more than twenty countries in more than fifteen currencies (Refinitiv, 2021). The highest country weight of this index is assigned to the USA, with 36.54%, closely followed by EGBI, refering to the countries of the Economic and Monetary Union (EMU) that fullfill the index's criteria, with 34.19% and Japan with 16.96% (Yieldbook, 2021).

5.3. Risk and return analysis

This chapter aims at presenting an in-depth analysis of the four indices used in this empirical analysis, focusing on their risk and return. In the risk and return analysis, the abovementioned four indices were compared in the period from 02.01.2004 to 01.01.2021 (seventeen years in total) in terms of their return and their volatility. Additionally, the correlation and covariance between these three asset classes were calculated.

5.3.1. Risk and return profile

When calculating the return of an asset with the assumption of a normal distribution of returns, the return of an asset are can be calculated by using a log-normal distribution of today´s and yesterday´s price, as reflected in the following formula (Choudhry, 2001, p. 652):

$$\ln\left(\frac{P_T}{P_{T-1}}\right)$$

Standard deviation is used to measure market volatility. Monthly standard deviation is calculated as the square root of the variance, which is expressed by the second moment of the deviation from the mean, as shown in the following formula (Burkett and Scherer, 2020, p.3):

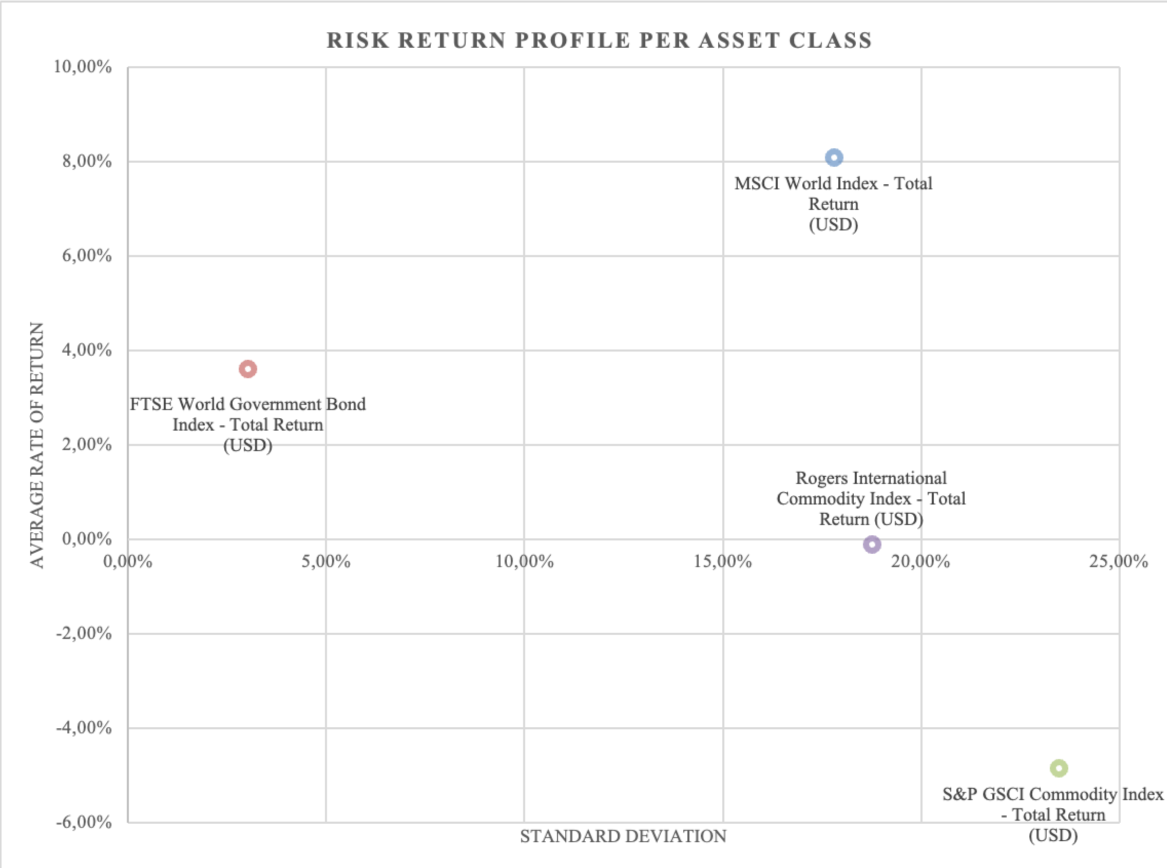
$$\sigma_M = \sqrt{\frac{1}{n}\sum_{i=1}^{n}(r_i - \bar{r})^2}$$

In this empirical analysis, weekly and annual rates of return of each individual index, as well as the standard deviation, were calculated based on weekly market prices during the time period of seventeen years (02.01.2004 to 01.01.2021). The annual rate of return and the standard deviation both help build a risk-return profile of the asset classes analysed.

The risk-return profile of the asset classes used in this empirical analysis (stocks, bonds and commodities), each represented by an index (see Sub-chapter 5.2), is presented in Figure 4 and Table 1.

Figure 4

Risk-return profile per asset class (02.01.2004 to 01.01.2021)



Note. Source: Own calculation.

Table 1

Risk-return profile of asset classes (02.01.2004 to 01.01.2021)

	MSCI World Index - Total Return (USD)	FTSE World Government Bond Index - Total Return (USD)	S&P GSCI Commodity Index - Total Return (USD)	Rogers International Commodity Index - Total Return (USD)
Average annual rate of return	8.09%	3.62%	-4.85%	-0.10%
Annual standard deviation	17.80%	3.03%	23.48%	18.76%

Note. Source: Own calculation.

In summary, as presented in Table 1, it can be concluded that the MSCI World Index - Total Return, a stock index, has the highest average annual rate of return of 8.09% over the period of time from 02.01.2004 to 01.01.2021, with a fairly high volatility. However, the highest volatility is associated with the commodity indices, the Rogers International Commodity Index - Total Return and the S&P GSCI Commodity Index - Total Return. The Rogers International Commodity Index - Total Return records a lower, but still negative average annual rate of return – as opposed to the other commodity index used in this empirical analysis, the S&P GSCI Commodity Index - Total Return, that records a much higher negative average annual rate of return of -4.85%, with the highest volatility among the analysed indices of 23.48% (see Table 1). The FTSE World Government Bond Index - Total Return, a bond index, recorded the lowest volatility among the analysed indices, while generating a higher average rate of return than both analysed commodity indices.

5.3.2. Correlation and covariance

In order to be able to calculate the three scenarios, in addition to risk and return profile of each asset class, the relationship between those asset classes needs to be taken into consideration as well. This is done by calculating the correlation and the covariance of these three asset classes. The covariance is defined as the level to which returns of two assets change together and can be positive (two assets changing in the same way) or negative (two assets changing in opposite ways), as outlined by Fabozzi and Grant (2001). The correlation is considered low when it is between 0 and 0.20, moderate when it is between 0.21 and 0.40 and high when it is higher than 0.40 (Das, 2003).

The covariance of a portfolio can be calculated by summing up how far removed the values x and y are from the mean and then dividing this with the observation number reduced by 1, as per Choudhry (2001, p.627):

$$Covariance = \frac{\sum_{i=1}^{n}(x_i - x)(y_i - y)}{n-1}$$

By dividing the covariance by the product of the volatilities (standard deviations), a comparable number is calculated, ranging from 1 (perfect correlation in the same direction) to -1 (perfect correlation in the opposite directions), called the correlation, as per Fabozzi and Grant (2001). Therefore, the correlation of a portfolio (ρ) can be calculated with the following formula, in which σ stands for the standard deviation of two variables (V_1, V_2) and cov stands for the covariance between two variables, as emphasized by Hull (2014, p.276):

$$\rho = \frac{cov(V_1, V_2)}{\sigma(V_1)\sigma(V_2)}$$

In order to calculate the correlation and the covariance of those asset classes, three scenarios were analysed. The variable, i.e. input data used for the correlation and covariance calculation in the empirical analysis, is the weekly rate of return of each index.

Scenario 1

In *Scenario 1*, the portfolio is composed exclusively of stocks and bonds, represented by the MSCI World Index - Total Return (for stocks) and the FTSE World Government Bond Index - Total Return (for bonds). As shown in Table 2 and Table 3, a negative moderate correlation and a negative covariance between the stock and the bond index is noted, i.e. the assets are changing in opposite ways, as outlined by Fabozzi and Grant (2001) and Das (2003).

Table 2

Correlation matrix for the MSCI World Index - Total Return and the FTSE World Government Bond Index - Total Return (02.01.2004 to 01.01.2021)

Correlation Matrix	MSCI World Index - Total Return (USD)	FTSE World Government Bond Index - Total Return (USD)
MSCI World Index - Total Return (USD)	1	-0.228213487
FTSE World Government Bond Index - Total Return (USD)	-0.228213487	1

Note. Source: Own calculation.

Table 3

Covariance matrix for the MSCI World Index - Total Return and the FTSE World Government Bond Index - Total Return (02.01.2004 to 01.01.2021)

Covariance Matrix	MSCI World Index - Total Return (USD)	FTSE World Government Bond Index - Total Return (USD)
MSCI World Index - Total Return (USD)	0.00060905	-0.00002366
FTSE World Government Bond Index - Total Return (USD)	-0.00002366	0.00001765

Note. Source: Own calculation.

Scenario 2

In *Scenario 2,* the S&P GSCI Commodity Index - Total Return is added to the portfolio, so that the portfolio consists of stocks, bonds and commodities – and is represented by the MSCI World Index - Total Return (for stocks), the FTSE World Government Bond Index - Total Return (for bonds) and the S&P GSCI Commodity Index - Total Return (for commodities). As shown in Table 4 and Table 5, the commodity index analysed in this scenario (S&P GSCI Commodity Index - Total Return) shows a low negative correlation and a negative covariance with the FTSE World Government Bond Index - Total Return (bonds) and a high positive correlation and a positive covariance with the MSCI World Index - Total Return (stocks), indicating that these two assets, the MSCI World Index - Total Return and the S&P GSCI Commodity Index - Total Return, change in the same way, as outlined by Fabozzi and Grant (2001) and Das (2003).

Table 4

Correlation matrix for the MSCI World Index - Total Return, the FTSE World Government Bond Index - Total Return and the S&P GSCI Commodity Index - Total Return (02.01.2004 to 01.01.2021)

Correlation Matrix	MSCI World Index - Total Return (USD)	FTSE World Government Bond Index - Total Return (USD)	S&P GSCI Commodity Index - Total Return (USD)
MSCI World Index - Total Return (USD)	1	-0.2282134869	0.4900468647
FTSE World Government Bond Index - Total Return (USD)	-0.2282134869	1	-0.1990830319
S&P GSCI Commodity Index - Total Return (USD)	0.4900468647	-0.1990830319	1

Note. Source: Own calculation.

Table 5

Covariance matrix for the MSCI World Index - Total Return, the FTSE World Government Bond Index - Total Return and the S&P GSCI Commodity Index - Total Return (02.01.2004 to 01.01.2021)

Covariance Matrix	MSCI World Index - Total Return (USD)	FTSE World Government Bond Index - Total Return (USD)	S&P GSCI Commodity Index - Total Return (USD)
MSCI World Index - Total Return (USD)	0.000609050	-0.000023662	0.000393725
FTSE World Government Bond Index - Total Return (USD)	-0.000023662	0.000017651	-0.000027230
S&P GSCI Commodity Index - Total Return (USD)	0.000393725	-0.000027230	0.001059880

Note. Source: Own calculation.

Scenario 3

In *Scenario 3*, the Rogers International Commodity Index - Total Return (RICI) is added to the portfolio from *Scenario 1*, so that the portfolio consists of stocks, bonds and commodities – and is represented by the MSCI World Index - Total Return (for stocks), the FTSE World Government Bond Index - Total Return (for bonds) and the Rogers International Commodity Index - Total Return (RICI) (for commodities). As shown in Table 6 and Table 7, the commodity index analysed in this scenario (Rogers International Commodity Index - Total Return) shows a low negative correlation and a negative covariance with the FTSE World Government Bond Index - Total Return (bonds) and a high positive correlation and a positive covariance with the MSCI World Index - Total Return (stocks), indicating that the MSCI World Index - Total Return and the Rogers International Commodity Index - Total Return change in the same way and the FTSE World Government Bond Index - Total Return and the Rogers International Commodity Index - Total Return change in opposite ways, as outlined by Fabozzi and Grant (2001) and Das (2003).

Table 6

Correlation matrix for the MSCI World Index - Total Return, the FTSE World Government Bond Index - Total Return and the Rogers International Commodity Index - Total Return RICI (02.01.2004 to 01.01.2021)

Correlation Matrix	MSCI World Index - Total Return (USD)	FTSE World Government Bond Index - Total Return (USD)	Rogers International Commodity Index - Total Return (USD)
MSCI World Index - Total Return (USD)	1	-0.228213487	0.539262691
FTSE World Government Bond Index - Total Return (USD)	-0.228213487	1	-0.207704937
Rogers International Commodity Total Return Index (USD)	0.539262691	-0.207704937	1

Note. Source: Own calculation.

Table 7

Covariance matrix for the MSCI World Index - Total Return, the FTSE World Government Bond Index - Total Return and the Rogers International Commodity Index - Total Return RICI (02.01.2004 to 01.01.2021)

Covariance Matrix	MSCI World Index - Total Return (USD)	FTSE World Government Bond Index - Total Return (USD)	Rogers International Commodity Index - Total Return (USD)
MSCI World Index - Total Return (USD)	0.000609050	-0.000023662	0.000346142
FTSE World Government Bond Index - Total Return (USD)	-0.000023662	0.000017651	-0.000022696
Rogers International Commodity Total Return Index (USD)	0.000346142	-0.000022696	0.000676481

Note. Source: Own calculation.

5.4. Data normality tests

Different statistical methods – like correlation, regression, variance etc. – assume data normality: that the mean is a good representative value for the data set used (Mishra et al., 2019). Therefore, the question of whether data normality can be assumed or not is crutial when analysing a data set.

Different normality tests of continuous data – such as the Kolmogorov-Smirnov test, the Shapiro-Wilk test, Q-Q Plot, histogram, skewness, kurtosis etc. – can be used (Mishra et al., 2019). The two most common methods, the Kolmogorov-Smirnov test and the Shapiro-Wilk test, as well as the Q-Q Plot, skewness, kurtosis and the histogram, are used within this seventeen-year empirical analysis, in order to test the normality of the data used, i.e. in this case, the indices′ weekly rates of return.

Additionally, as per Mishra et al. (2019), the mean (i.e. the average), the median (the middle observation with data that is increasing or decreasing), minimum and maximum and the 1^{st} and 3^{rd} Quartile for the data used (the weekly returns of all four total return indices used in this empirical analysis) were calculated in RStudio (see Appendix 1). This is presented in Table 8 below:

Table 8

Mean, Median, Minimum, Maximum, 1st and 3rd Quartile calculation for the four indices used in the empirical analysis (02.01.2004 to 01.01.2021)

	Min.	1st Quartile	Median	Mean	3rd Quartile	Max.
MSCI World Index - Total Return (USD)	-0.223341	-0.008201	0.00325	0.001556	0.013408	0.116981
FTSE World Government Bond Index - Total Return (USD)	-0.0226177	-0.0018468	0.0009504	0.0006957	0.003363	0.0157814
S&P GSCI Commodity index - Total Return (USD)	-0.211347	-0.0175951	0.0015446	-0.0009325	0.0196666	0.1213192
Rogers International Commodity Index - Total Return (USD)	-1.813E-01	-1.304E-02	9.849E-04	-1.982E-05	1.543E-02	1.014E-01

Note. Source: Own calculation.

5.4.1. Histogram

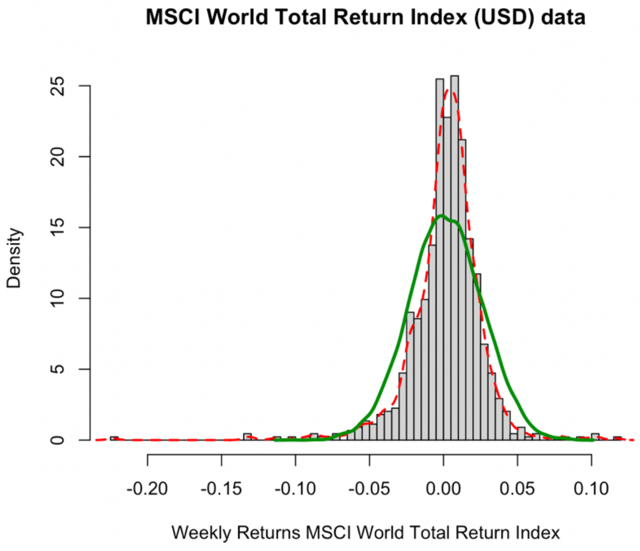
A histogram is an approximation of the continuous data´s probability distribution; if bell-shaped and symmetrically distributed around the mean, the data is considered normally distributed (Mishra et al., 2019). The histograms of the weekly rates of return of the four indices used in this empirical analysis were drawn in RStudio (see Appendix 1). The density curve in form of a bell is known as the normal standard distribution and is defined by the mean and the standard deviation (Mishra et al., 2019). The green line in the histograms below depicts the normal standard distribution, while the red line represents the Kernel density of the empirical data used. The Kernel density estimation is used to estimate the probability density function from the data available – in this case, the weekly rates of return during the observed time period (Węglarczyk, 2018). Rachev, Menn and Fabozzi (2005) argued that extreme price fluctuations can be seen in the distributions´ “tails“ (for example, economy crashes or booms).

MSCI World Index - Total Return (USD)

Histograms of the weekly rates of return of the MSCI World Index - Total Return (USD), for the time period 02.01.2004 - 01.01.2021, are presented in Figure 5 and Figure 6.

Figure 5

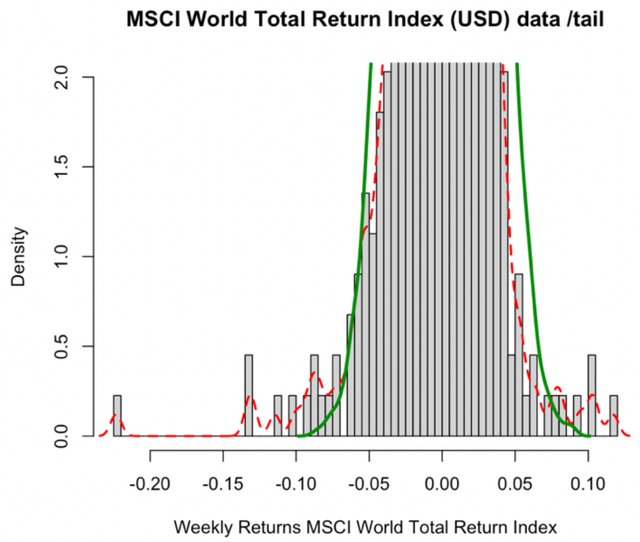
Histogram of the MSCI World Index - Total Return (USD)



Note. Source: Own calculation.

Figure 6

Histogram of the MSCI World Index - Total Return (USD)/ tail



Note. Source: Own calculation.

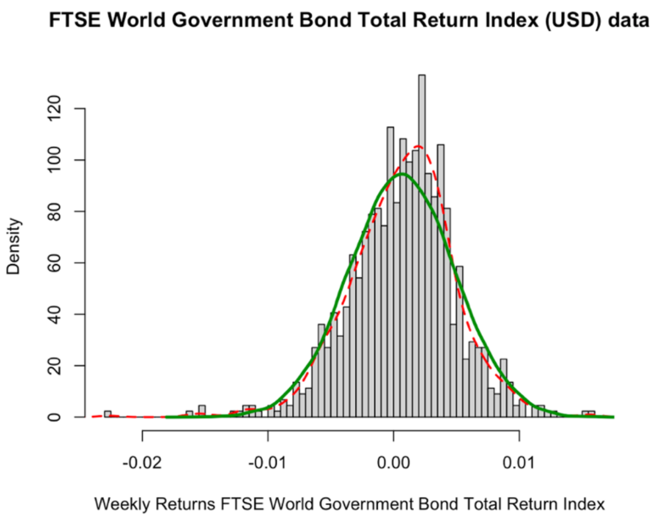
The Kernel density estimation of the weekly rates of return of the MSCI World Index - Total Return, marked with the red line in in Figure 5 and Figure 6, depicts fatter tails, compared to the normal distribution, which is marked with the green line. These fatter tails are making the occurance of extreme events (for example, economy crashes or booms) more likely, compared to the normal distribution (Rachev, Menn and Fabozzi, 2005).

FTSE World Government Bond Index - Total Return (USD)

For the time period 02.01.2004 to 01.01.2021, histograms of the weekly rates of return of the FTSE World Government Bond Index - Total Return (USD), are presented in Figure 7 and Figure 8.

Figure 7

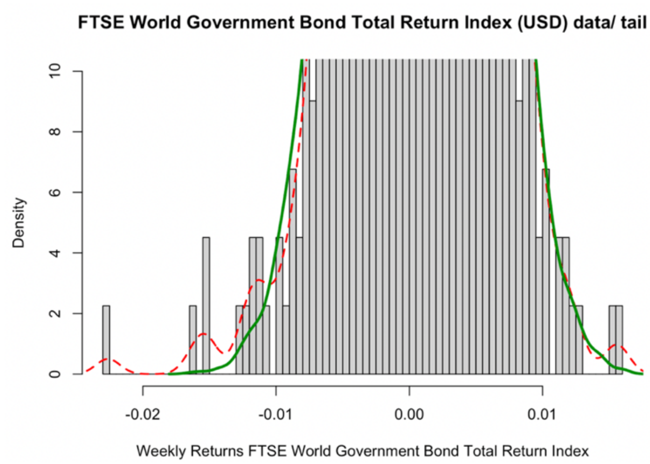
Histogram of the FTSE World Government Bond Index - Total Return (USD)



Note. Source: Own calculation.

Figure 8

Histogram of the FTSE World Government Bond Index - Total Return (USD)/ tail



Note. Source: Own calculation.

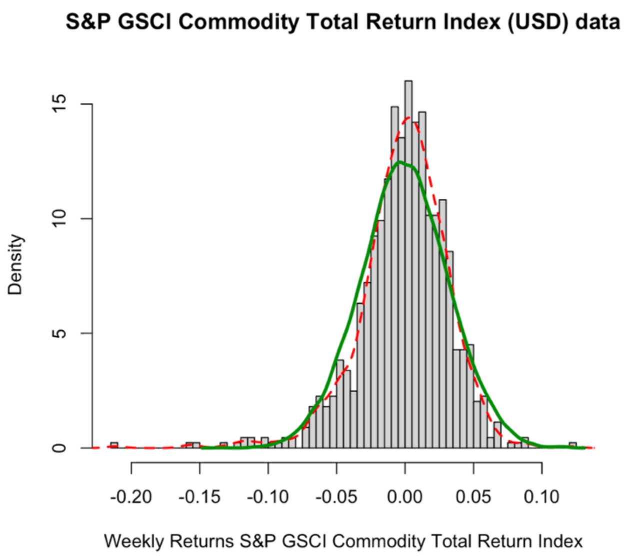
Due to a few outliers in the tails and the skewness of the Kernel density estimation of the weekly rates of return of the FTSE World Government Bond Index - Total Return, marked with the red line in Figure 7 and Figure 8, there is no overlap with the normal distribution, marked with the green line. However, due to less heavy tails than observed in the other three indices analysed, occurance of extreme events (for example, economy crashes or booms) is less likely in comparison with the normal distribution (Rachev, Menn and Fabozzi, 2005).

S&P GSCI Commodity Index - Total Return Index (USD)

Histograms of the weekly rates of return of the S&P GSCI Commodity Index - Total Return (USD) for the time period 02.01.2004 to 01.01.2021 are presented in Figure 9 and Figure 10.

Figure 9

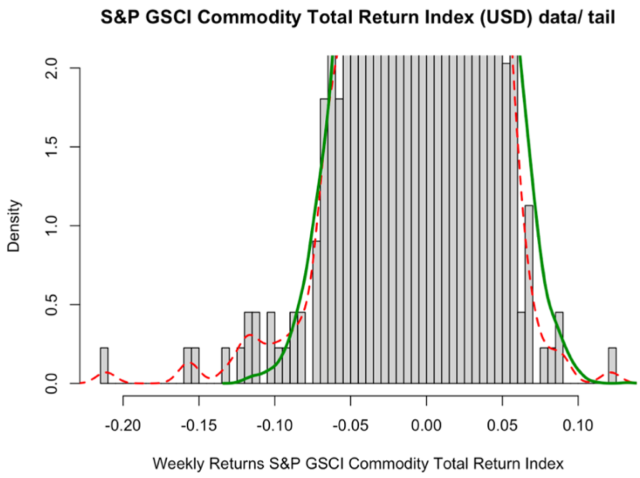
Histogram of the S&P GSCI Commodity Index - Total Return (USD)



Note. Source: Own calculation.

Figure 10

Histogram of the S&P GSCI Commodity Index - Total Return (USD)/ tail



Note. Source: Own calculation.

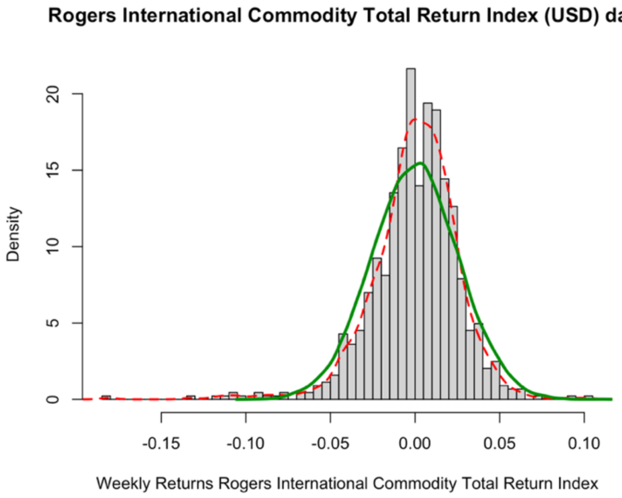
In the histogram above, the Kernel density estimation of the weekly rates of return of the S&P GSCI Commodity Index - Total Return is marked with the red line in Figure 9 and Figure 10 and it portrays fatter tails than the normal distribution, marked with the green line, therefore making the occurance of extreme events (for example, economy crashes or booms) more likely compared to the normal distribution (Rachev, Menn and Fabozzi, 2005).

Rogers International Commodity Index - Total Return (USD)

For the time period 02.01.2004 to 01.01.2021, histograms of the weekly rates of return of the Rogers International Commodity Index - Total Return (RICI) are presented in Figure 11 and Figure 12.

Figure 11

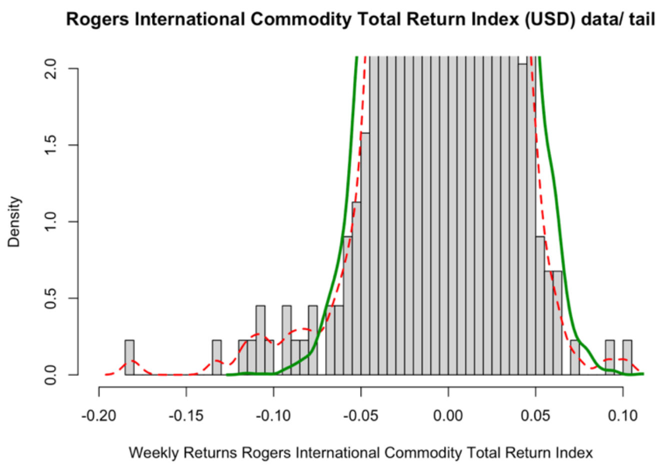
Histogram of the Rogers International Commodity Index - Total Return (USD)



Note. Source: Own calculation.

Figure 12

Histogram of the Rogers International Commodity Index - Total Return (USD)/ tail



Note. Source: Own calculation.

The red line of the Kernel density estimation of the weekly rates of return of the Rogers International Commodity Index - Total Return in Figure 11 and Figure 12 portrays fatter tails than the normal distribution, making the occurance of extreme events (for example, economy crashes or booms) more likely compared to the normal distribution (Rachev, Menn and Fabozzi, 2005).

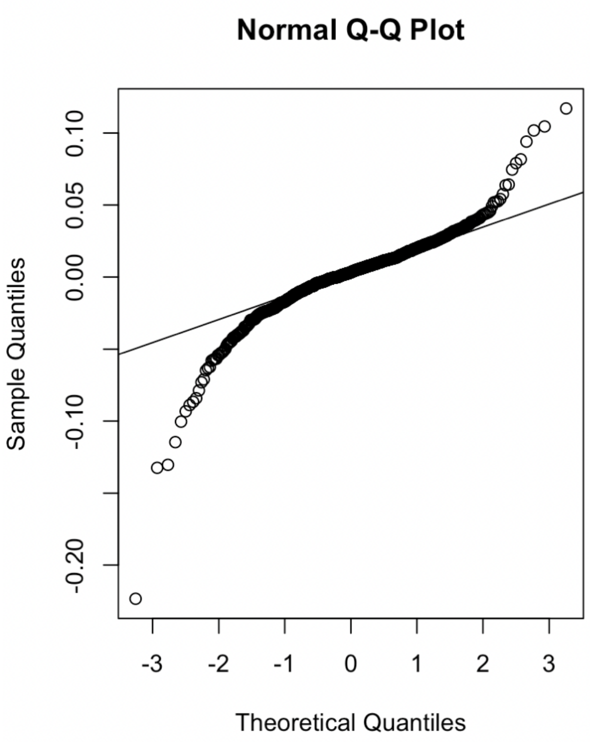
5.4.2. Q-Q Plot

In the Quantile-Quantile Plot (Q-Q Plot), two quantiles, the observed and the expected one, are plotted against each other and if the observed data and the expected data overlap, the data is assumed to be normally distributed (Mishra et al., 2019). Q-Q Plots of weekly rates of return of all four indices analysed in this empirical analysis, over a time period of seventeen years (02.01.2004 to 01.01.2021), were drawn in RStudio (see Appendix 1) and are presented below.

MSCI World Index - Total Return (USD)

Figure 13

Normal Q-Q Plot of the MSCI World Index - Total Return (USD)



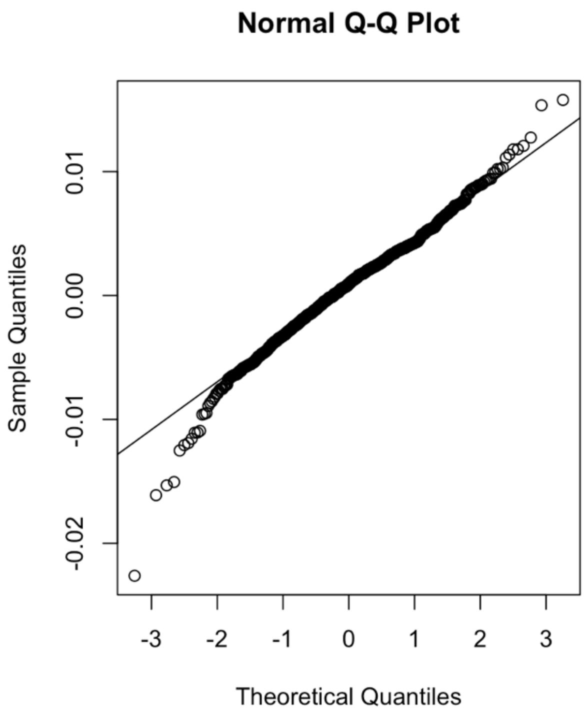
Note. Source: Own calculation.

In case of the MSCI World Index - Total Return, the observed data and the expected data do not overlap, the data is therefore not assumed to be normally distributed, which is consistent with the conclusions in Sub-chapter 5.4.1 (Mishra et al., 2019).

FTSE World Government Bond Index - Total Return (USD)

Figure 14

Normal Q-Q Plot of the FTSE World Government Bond Index - Total Return (USD)



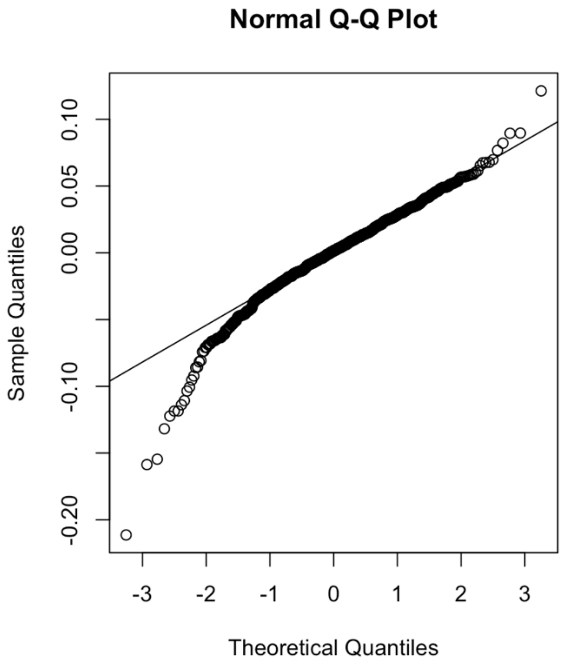
Note. Source: Own calculation.

While analysing the normal Q-Q Plot of the FTSE World Government Bond Index - Total Return it can be concluded that the observed data and the expected data do not overlap, and the data is therefore not assumed to be normally distributed, which is consistent with the conclusions in Sub-chapter 5.4.1 (Mishra et al., 2019).

S&P GSCI Commodity Index - Total Return Index (USD)

Figure 15

Normal Q-Q Plot of the S&P GSCI Commodity Index - Total Return (USD)



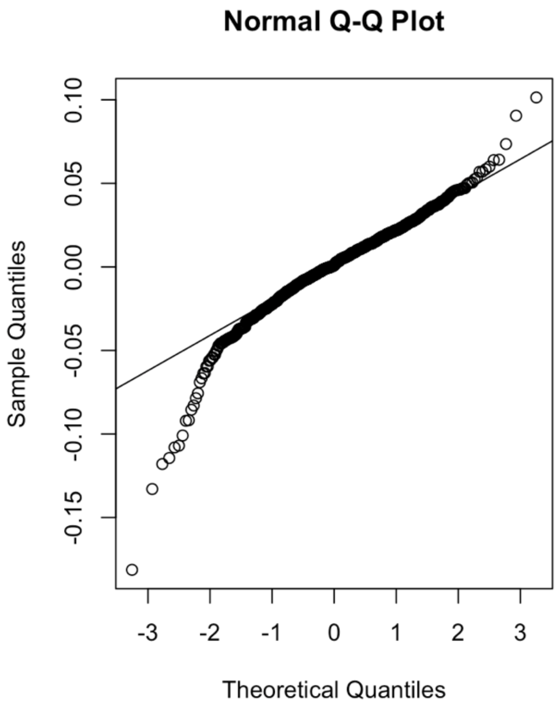
Note. Source: Own calculation.

The normal Q-Q Plot of the S&P GSCI Commodity Index - Total Return indicates that the observed data and the expected data do not overlap, and the data is therefore not assumed to be normally distributed, which is consistent with the conclusions in Sub-chapter 5.4.1 (Mishra et al., 2019).

Rogers International Commodity Index - Total Return (USD)

Figure 16

Normal Q-Q Plot of the Rogers International Commodity Index - Total Return (USD)



Note. Source: Own calculation.

The observed data and the expected data do not overlap in the case of the Rogers International Commodity Index - Total Return, and the data is therefore not assumed to be normally distributed, which is consistent with the conclusions in Sub-chapter 5.4.1 (Mishra et al., 2019).

5.4.3. Skewness and kurtosis

In addition to the risk and return profile of each asset class and their correlation and covariance, the skewness and kurtosis of each asset class was calculated as well and is presented in this chapter.

Skewness and kurtosis are higher normal distribution moments - the former being the third and the latter being the fourth and they can be expressed with the following formulas, according to Choudhry (2001, p.654):

$$Skewness = \frac{\frac{1}{n}\sum_{i=1}^{n}(x_i - \bar{x})^3}{\sigma^3}$$

$$Kurtosis = \frac{\frac{1}{n}\sum_{i=1}^{n}(x_i - \bar{x})^4}{\sigma^4}$$

Therefore, skewness represents the difference between the mean and the mode, divided by the standard deviation (Jambu, 1991). The asymmetry from normal distribution is expressed through skewness, and it can be either positive (skewness to the right) or negative (skewness to the left), as implied by Anuwoje and Togborlo (2019).

The skewness of assets is highly important to investors when deciding on future investments, being that positive skewness is linked with possibly higher average returns of an asset, since in these cases the mean and the median are higher than the mode (Anuwoje and Togborlo, 2019). On the other hand, negative skewness (skewness to the left) is linked to the mean and the median being beneath the mode (Anuwoje and Togborlo, 2019). However, according to Francis and Kim (2013), which of the abovementioned central tendency measures serve as the investor´s measures of return is not specified. Be that as it may, by analysing skeweness of the data, investors can make a better informed prediction about future outcomes of the investment; companies that are positively skewed are very attractive for investment, due to the possibility of gaining greater average returns in the future, while companies with negative skeweness are less attractive for investmenrs due to the possibility of higher future losses (Anuwoje and Togborlo, 2019).

The skewness and kurtosis of the four indices used in this empirical analysis, the MSCI World Index - Total Return , the FTSE World Government Bond Index - Total Return , the S&P GSCI Commodity Index - Total Return and the Rogers International Commodity Index - Total Return, were calculated in Microsoft Excel and verified in RStudio. Weekly rate of returns of the four abovementioned indices were used as input data for the skewness and kurtosis calculation.

Table 9

Skewness and kurtosis of the MSCI World Index - Total Return (USD) and the FTSE World Government Bond Index - Total Return (USD)

MSCI World Index - Total Return (USD)		FTSE World Government Bond Index - Total Return (USD)	
Skewness	Kurtosis	Skewness	Kurtosis
-1.419214741	11.8938747	-0.394014158	1.829262074

Note. Source: Own calculation.

Table 10

Skewness and kurtosis of the S&P GSCI Commodity Total Return Index (USD) and the Rogers International Commodity Total Return Index (USD)

S&P GSCI Commodity Index - Total Return (USD)		Rogers International Commodity Total Return Index (USD)	
Skewness	**Kurtosis**	**Skewness**	**Kurtosis**
-0.950371725	3.791606564	-1.071769071	5.135082492

Note. Source: Own calculation.

As shown in Table 9 and Table 10, all four indices analysed record negative skewness, making them less attractive for investments, due to the possibility of higher future losses (Anuwoje and Togborlo, 2019). As presented in Table 8, all four indices analysed have a higher median than the mean, also indicating a negative skewness of the four abovementioned indices (Francis and Kim, 2013). The weekly rates of return of all four indices analysed deviate from normal distribution and since they are negatively skewed, the data is skewed to the left (Anuwoje and Togborlo, 2019).

Kurtosis expresses the probability of an extreme event arising with regard to a particular distribution and can be higher than 3 (for future returns with higher volatility, i.e. higher probability of extremely high or extremely low future returns) or lower than 3, since 3 is the kurtosis of standard normal distribution and is used as a benchmark (Anuwoje and Togborlo, 2019). As presented in Table 9 and Table 10, the MSCI World Index - Total Return, the S&P GSCI Commodity Index - Total Return and the Rogers International Commodity Index - Total Return record a kurtosis higher than 3, indicating higher probability of extremely high or extremely low future returns with these indices, as argued by Anuwoje and Togborlo (2019). The FTSE World Government Bond Index - Total Return, however, records a kurtosis lower than 3, meaning that there is no higher probability of extremely high or extremely low future returns with this index (Anuwoje and Togborlo, 2019).

5.4.4. Kolmogorov-Smirnov test

While the Kolmogorov-Smirnov test is usually used for larger samples (n>=50), the Shapiro-Wilk test is usually the first choice for small samples (n<50), even though it can be used on large samples as well (Mishra et al., 2019). In both cases, the null hypothesis assumes that the data is drawn from a population that is normaly distributed. If the significance level is higher than 0.05 (P>0.05), the data is viewed as normally distributed and the null hypothesis, therefore, is accepted (Mishra et al., 2019).

For the purpose of this empirical analysis, both tests, the Kolmogorov-Smirnov test and the Shapiro-Wilk test, were calculated in RStudio based on the weekly rate of return of the indices (see Appendix 1).

The results of the Kolmogorov-Smirnov test for all four indices, the MSCI World Index - Total Return, the FTSE World Government Bond Index - Total Return, the S&P GSCI Commodity Index - Total Return and the Rogers International Commodity Index - Total Return (RICI), show that the p-value is lower than 2.2e-16. Since the significance level is lower than 0.05, the null hypothesis, stating that the population is distributed normally, can be rejected in all four cases (Mishra et al., 2019).

5.4.5. Shapiro-Wilk test

Since the Shapiro-Wilk test can be used on large samples as well, this test was also calculated for the weekly rates of return of all four indices used in this empirical analysis (Mishra et al., 2019).

The results of the Shapiro-Wilk test for the MSCI World Index - Total Return and the Rogers International Commodity Index - Total Return (RICI) show the same image as the Kolmogorov-Smirnov test. The results of the Shapiro-Wilk tests show a p-value lower than 2.2e-16. Therefore, the significance level in this case as well is lower than 0.05 and the null hypothesis, stating that the population is distributed normally, can be rejected in both cases as well (Mishra et al., 2019).

Since the results of the Shapiro-Wilk test show a p-value of 1.174e-08 for the FTSE World Government Bond Index - Total Return and a p-value of 5.384e-16 for the S&P GSCI Commodity Index - Total Return, the significance level is lower than 0.05 in both cases and the null hypothesis, stating that the population is distributed normally, is rejected in both cases as well (Mishra et al., 2019).

5.5. Portfolio optimization

The aim of this part of the seventeen-year empirical analysis is to select the *optimal portfolio* by using the Markowitzs mean-variance model. The optimal portfolio is the one an investor chooses among all available portfolios that brings the investor the maximum expected return while remaining in compliance with the said investor's risk appetite (i.e. the portfolio chosen among all available efficient portfolios), as argued by Fabozzi and Grant (2001).

Portfolio optimization was calculated in this empirical analysis with the use of a Markowitzs mean-variance model in Microsoft Excel. For the purpose of this empirical analysis, short-selling was excluded from the calculation – asset weights were assumed to be positive, i.e. weights in every asset class are assumed to be greater than or equal to zero (Cheung and Miu, 2010).

Three different portfolio optimization calculations, i.e. different scenarios, were assumed and their results analysed as a part of this empirical analysis:

Scenario 1

In the first scenario, the portfolio is composed exclusively of stocks and bonds. The MSCI World Index - Total Return is used for stock return movements and the FTSE World Government Bond Index - Total Return is used for bond return movements.

Scenario 2

In the second scenario, the S&P GSCI Commodity Index - Total Return is added to the portfolio of stocks and bonds, now consisting of stocks, bonds and commodities.

Scenario 3

In the third scenario, the Rogers International Commodity Index - Total Return (RICI) is added to the portfolio, in place of the S&P GSCI Commodity Index - Total Return, as a representative asset for commodities, so that the portfolio consists of stocks, bonds and a different commodity when compared to *Scenario 2*.

In addition to the risk-return profiles of each randomly assumed portfolio, Sharpe ratios and Value at Risk (VaR) were calculated and analysed as well as compared between all scenarios.

5.5.1. Input and output data of portfolio optimization

In order to choose the optimal portfolio by using the Markowitzs mean-variance model, following indicators for each of the two thousand randomly weighted portfolios were calculated:

Portfolio´s variance and standard deviation

According to Francis and Kim (2013, p. 27), in a two-asset portfolio, the variance of returns is calculated with the following formula, in which w represents the weight and σ^2 represents the variance of each asset:

$$\sigma_p^2 = w_1^2\sigma_1^2 + w_2^2\sigma_2^2 + 2w_1w_2\sigma_{12}$$

In a three-asset portfolio, the formula calculating the variance of returns is changed in the following manner (Francis and Kim, 2013, p. 28):

$$\sigma_p^2 = w_1^2\sigma_1^2 + w_2^2\sigma_2^2 + w_3^2\sigma_3^2 + 2w_1w_2\sigma_{12} + 2w_2w_3\sigma_{23} + 2w_1w_3\sigma_{13}$$

In a portfolio with n assets, this formula of portfolio variance of returns can be expressed differently, so that σ_{ij} represents the covariance between assets´ returns i and j, and w represents the weights of the asset within the portfolio (Francis and Kim, 2013, p. 26):

$$\sigma_p^2 = \sum_{i=1}^{n}\sum_{j=1}^{n} w_iw_j\sigma_{ij}$$

Input data for calculating the variance and the standard deviation entails (1) randomly assumed weights per asset in the portfolio and (2) the covariance matrix derived from assets´ weekly rates of returns. The calculation was done using array functions, MMULT and TRANSPOSE in Microsoft Excel (Jackson and Staunton, 2001).

In order to calculate the optimal portfolio, two thousand random asset weights were generated in Microsoft Excel. However, excluded from the calculation was short-selling, i.e. asset weights were assumed to be positive, as per Cheung and Miu (2010), and therefore it was assumed that the sum of each portfolio weight equals one. Since the covariance matrix used for the calculation of the variance was based on the indices´ weekly rates of returns, the standard deviation was annualized in order to proceed with the further calculations with annualized data.

Expected return of the portfolio

The expected return of a portfolio is calculated with the following formula, in which $E(r_i)$ represents the expected return of the *i*th asset and w_i represents its weight (Jackson and Staunton, 2001, p.103):

$$E(r_p) = \sum w_i\, E(r_i)$$

Therefore, the weights of each asset within the portfolio and the average annual rate of return per asset, were used as input data for calculating the expected return of each of the two thousand randomly chosen portfolios. The average annual rate of return per asset was calculated as an average from the weekly rates of returns, which were calculated by using a log-normal distribution of this Friday´s and last Friday`s price and then annualizing the rate of return (Choudhry, 2001). The calculation was done using array functions, SUMPRODUCT in Microsoft Excel (Jackson and Staunton, 2001).

5.5.2. Sharpe ratio and Value at Risk (VaR)

Sharpe ratio

The Sharpe ratio, first introduced by William F. Sharpe, is a risk-adjusted return measure that changes with the changing weights of the portfolio assets and it indicates how good the return of the assets counterbalances the risk the investor has undertaken (Anuwoje and Togborlo, 2019). Therefore, the Sharpe ratio (S_p) is calculated with the following formula, in which R_p stands for the average portfolio return, R_f for the risk-free rate and σ_p for the standard deviation of the portfolio (Anuwoje and Togborlo, 2019, p.33):

$$S_p = \frac{R_p - R_f}{\sigma_p}$$

Superior investment opportunities are linked to higher Sharpe ratios as they are calculated as the average return of the portfolio minus the risk-free rate, divided by the standard deviation of the portfolio, as per Anuwoje and Togborlo (2019).

Since the portfolio´s standard deviation and expected return for each of the two thousand randomly chosen portfolios were calculated earlier, these values were used as input data for the Sharpe ratio calculations per portfolio. As the risk-free rate of return, for the purpose of this empirical analysis, a three-month treasury bill per 31.12.2020, amounting to 0.09%, was used (U.S. Department of the Treasury, 2021).

The Sharpe ratio will be used in selecting the *optimal portfolio* according to the Markowitzs mean-variance model, since the *optimal portfolio* is the portfolio with the highest Sharpe ratio (Francis and Kim, 2013).

Value at Risk (VaR)

Additionally, Value at Risk (VaR) was calculated for all three scenarios and its results compared and analysed, since it is a very helpful financial risk assessment tool for portfolios and very popular with portfolio managers for portfolio risk management (Francis and Kim, 2013).

According to Hull (2014), when calculating the Value at Risk (VaR), one is calculating the maximum amount of money one might lose in a certain period of time with a given probability. Therefore, the Value at Risk (VaR) is a function of the following parameters: time period and the confidence interval (Hull, 2014).

Value at Risk is calculated using the following formula, in which μ_i represents the expected return, z_α equals 2.326 in a 99% confidence level and 1.645 in a 95% confidence level and σ_i is the standard deviation of returns (Francis and Kim, 2013, p.262):

$$VaR = \mu_i - z_{\alpha*}\sigma_i$$

For the purpose of this empirical analysis, the Value at Risk (VaR) is calculated for each portfolio under the following assumptions: confidence level of 99% and an investment of 1.000.000 USD. Since the portfolio´s standard deviation and expected return for each of the two thousand randomly selected portfolios were calculated earlier, these values were used as input data for the VaR calculations per portfolio. Since it is assumed that the confidence level is 99%, z_α equals 2.326 for the purpose of this empirical analysis (Francis and Kim, 2013).

5.5.3. Portfolio optimization (Scenario 1)

The first scenario is characterized by a portfolio composed exclusively of stocks (with stock price movements represented by the MSCI World Index - Total Return) and bonds (with bond price movements represented by the FTSE World Government Bond Index - Total Return).

In order to calculate the portfolio optimization, two thousand random asset weights are generated, with the sum of each portfolio equaling one. The average rate of return calculated for the stock and bond indices (MSCI World Index - Total Return and FTSE World Government Bond Index – Total Return) in Sub-chapter 5.3.1 is used to calculate the expected return per random assumed portfolio. In addition, the variance (and thus also the standard deviation) per assumed asset weight is calculated by using the previously generated covariance matrix calculated in Sub-chapter 5.3.2. The Sharpe ratios and Value at Risk (VaR) of each portfolio in *Scenario 1* are calculated as well.

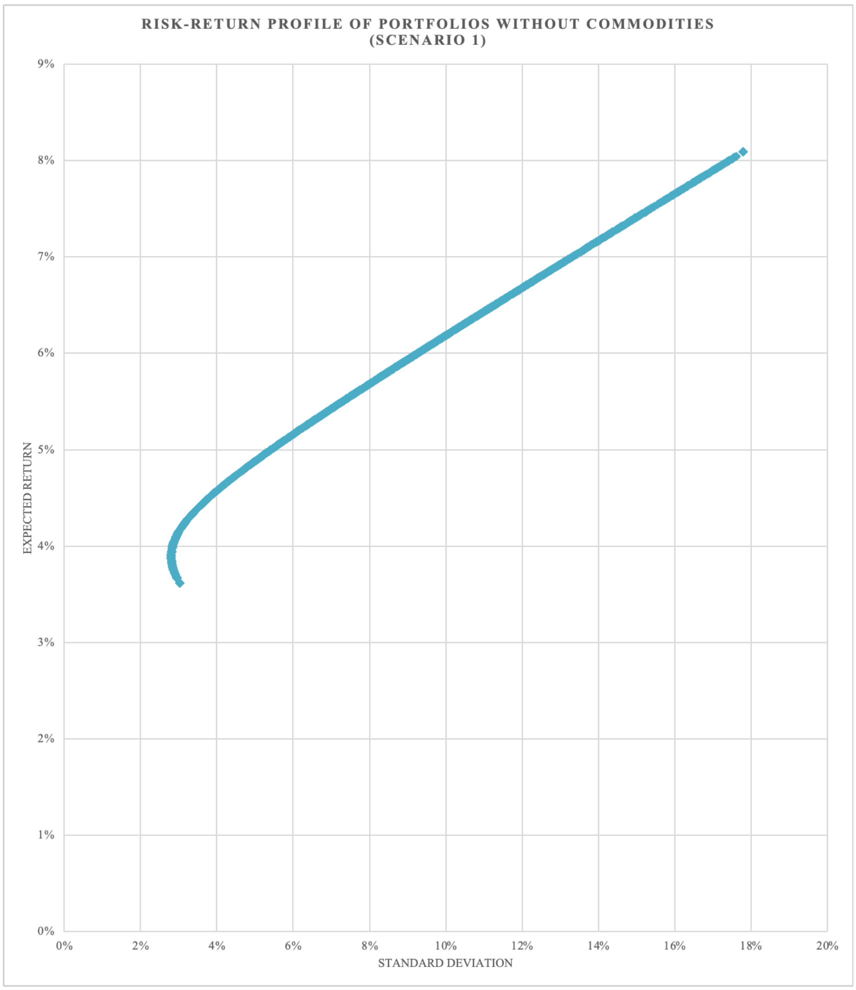
Risk-Return Profile

After calculating both the expected return and the standard deviation per assumed asset class within each portfolio, as described in Sub-chapter 5.5.1, a risk-return profile graphic of portfolios composed exclusively of stocks (represented by weekly returns of the MSCI World Index - Total Return) and bonds (represented by weekly returns of the FTSE World Government Bond) can be plotted.

The risk-return profile of *Scenario 1* is presented in Figure 17:

Figure 17

Risk-return profile of portfolios without commodities in Scenario 1 (02.01.2004 to 01.01.2021)



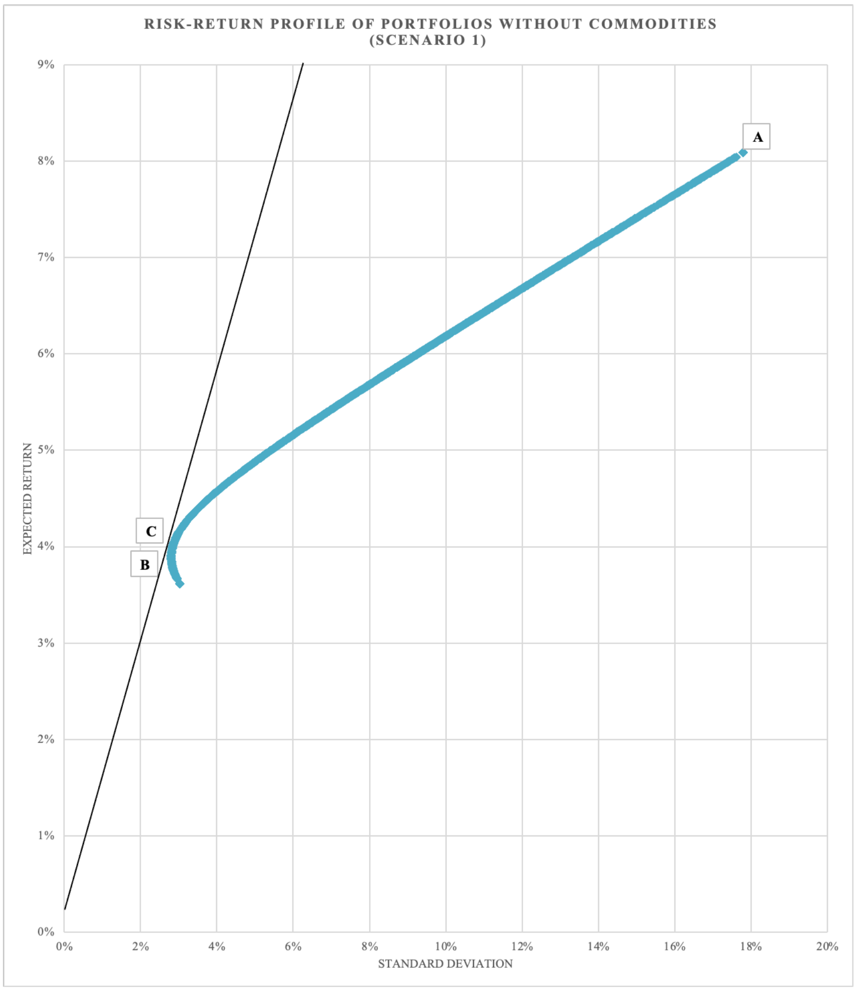
Note. Source: Own calculation.

This graphic shows a variety of investment options when it comes to these two indices (MSCI World Index - Total Return and FTSE World Government Bond Index – Total Return), all differing from one another by the assets' weight in the portfolio.

In Figure 17, three points of interest were identified: the point with the maximum expected return (point A), the point with the minimum standard deviation (point B) and the point with the maximum Sharpe ratio (point C), due to its connection with superior investment opportunities, as per Anuwoje and Togborlo (2019). The risk-return profile od *Scenario 1* showing the points A to C, is presented in Figure 18.

Figure 18

Risk-return profile of portfolios without commodities (Scenario 1) showing the portfolios A to C (02.01.2004 to 01.01.2021)



Note. Source: Own calculation.

Portfolios of points A to C, with their expected returns and standard deviations, have the following composition of indices (see Table 11):

Table 11

Weights, expected returns and standard deviations of points A to C from Figure 18

Point	Description	Weights MSCI World Index - Total Return (USD)	Weights FTSE World Government Bond Index - Total Return (USD)	Expected return	Standard Deviation
A	**Max. Expected Return**	100.00%	0.00%	8.09%	17.80%
B	**Min. Standard Deviation**	6.12%	93.88%	3.89%	2.80%
C	**Max. Sharpe Ratio**	8.70%	91.30%	4.01%	2.84%

Note. Source: Own calculation.

Within two thousand randomly generated portfolios in *Scenario 1*, where the portfolio consists of stock and bond indices, the portfolio with 100% invested in the stock index, the MSCI World Index - Total Return and 0% invested in the bond index, the FTSE World Government Bond Index - Total Return, had the maximum expected return. The portfolio with the minimum standard deviation was the one in which the majority invested was in the bond index, the FTSE World Government Bond Index - Total Return, with 93.88%, while 6.12% were invested in the stock index, the MSCI World Index - Total Return. Since the highest Sharpe ratio is linked to superior investment opportunities, this portfolio was analysed as well (Anuwoje and Togborlo, 2019). The majority of the portfolio with the highest Sharpe ratio is invested in the bond index, i.e. the FTSE World Government Bond Index - Total Return, with 91.30%, and 8.70% in the stock index, the MSCI World Index - Total Return. These findings confirm those from Sub-chapter 5.3.1, since the MSCI World Index - Total Return, a stock index, had a higher average rate of return than the FTSE World Government Bond Index - Total Return, a bond index, which recorded a much lower volatility than the stock index, which surely influences the way the weights are distributed between the assets within the portfolio (see Table 11).

Value at Risk

The minimum VaR in this scenario amounts to 25,956.96 USD, while the maximum VaR amounts to 333,038.13 USD. The portfolio composition with the minimum VaR is as follows: MSCI World Index - Total Return (representing stocks) weighting 7.84% and FTSE World Government Bond (representing bonds) weighting 92.16%. The minimum VaR indicates that this portfolio composition might be more favorable to invest in, comparatively speaking, due to the lowest amount of money one might lose when investing in it. On the other hand, the portfolio composition with the maximum VaR (MSCI World Index - Total Return weighting 100% and FTSE World Government Bond weighting 0%) is the least favorable investment option, since it may result in the highest amount of money being lost Hull (2014).

Sharpe Ratio

The highest Sharpe ratio, and therefore the superior investment option in this scenario, amounts to 1.37685128, having the following portfolio composition: MSCI World Index - Total Return (representing stocks) weighting 8.70% and FTSE World Government Bond (representing bonds) weighting 91.20%. On the other hand, the portfolio composed 100% of the MSCI World Index - Total Return and 0.00% of the FTSE World Government Bond has the lowest Sharpe ratio of 0.449546985, and is therefore the least favorable investment option in this scenario (Anuwoje and Togborlo, 2019).

Markowitz efficient portfolios are linked to creating portfolios with the highest expected return with a given variance, as stated by Fabozzi and Grant (2001). Since the optimal portfolio, the best choice on the Markowitz efficient frontier, is the one where the utility function touches the Markowitz efficient frontier and the utility function is linked to investors' preferences, different preferences may lead to different utility functions and – as a result – to a different optimal portfolio altogether (Fabozzi and Grant, 2001). Problems might occur when trying to quantify this utility function, as no instructions on how to measure one exactly even exist and it is therefore up to investors to decide which portfolio on the Markowitz efficient frontier is the optimal portfolio for them, given their individual risk and return preferences, as per Fabozzi and Grant (2001).

Due to difficulties when it comes to quantifying the utility functions when deciding on the optimal portfolio on the Markowitz efficient frontier, the addition of a risk-free rate (R_f) helps choose the right portfolio on the frontier. The portfolio that should be chosen in the *tangency portfolio* (i.e. market portfolio) is the one where the steepest line, called the *Capital Market Line* (CML), touches the efficient frontier, as outlined in Sub-chapter 3.1.4 (Schulmerich, 2013). Therefore, the aim is to maximize the Sharpe ratio, being that the Sharpe ratio represents the excess of portfolio return over the risk-free rate (R_f), divided by portfolio volatility (Schulmerich, 2013).

As outlined in the section above, the maximum Sharpe ratio in this empirical analysis amounts to 1.37685128 and is presented as the point C (the portfolio composing of 8.70% of the MSCI World Index - Total Return and 91.20% of the FTSE World Government Bond) in Figure 9, making it the tangency portfolio (i.e. market portfolio or optimal portfolio).

5.5.4. Portfolio optimization (Scenario 2)

The approach in this scenario resembles the one in *Scenario 1*. However, *Scenario 2* entails a portfolio consisting of three instead of two asset classes. In this scenario, the portfolio is composed of stocks (with stock price movements represented by the MSCI World Index - Total Return), bonds (bond price movements represented by the FTSE World Government Bond Index – Total Return) and commodities (with commodity price movements represented by the S&P GSCI Commodity Index - Total Return).

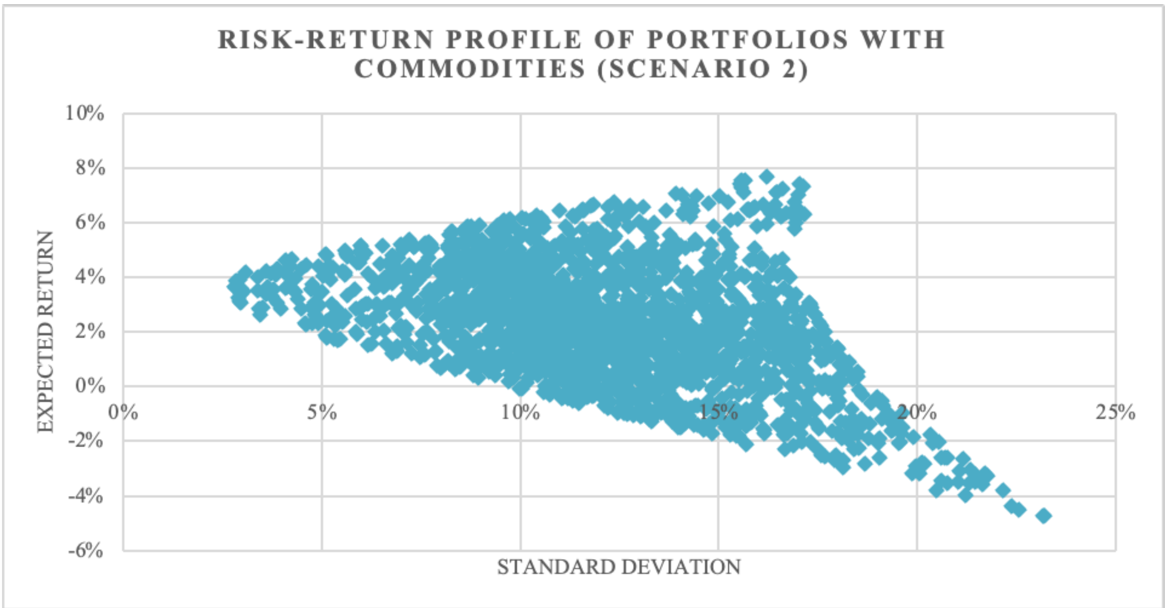
In order to calculate the portfolio optimization in *Scenario 2*, two thousand random asset weights are generated, just like in *Scenario 1* – with the sum of each portfolio equaling one. The average rate of return calculated in Sub-chapter 5.3.1 for the stock, bond and commodity indices (MSCI World Index - Total Return, FTSE World Government Bond and S&P GSCI Commodity Index - Total Return) is used to calculate the expected return per random assumed portfolio. The variance calculation (and therefore also the standard deviation) of each asset within the portfolio is calculated by using the previously generated covariance matrix from Sub-chapter 5.3.2. In addition, Sharpe ratio and Value at Risk (VaR) of each portfolio in *Scenario 2* are calculated as well.

Risk-Return Profile

A risk-return profile for *Scenario 2* of portfolios composed of stocks (represented by the MSCI World Index - Total Return), bonds (represented by the FTSE World Government Bond Index – Total Return) and commodities (represented by the S&P GSCI Commodity Index - Total Return) is shown in Figure 19.

Figure 19

Risk-return profile of portfolios with commodities in Scenario 2 (02.01.2004 to 01.01.2021)



Note. Source: Own calculation.

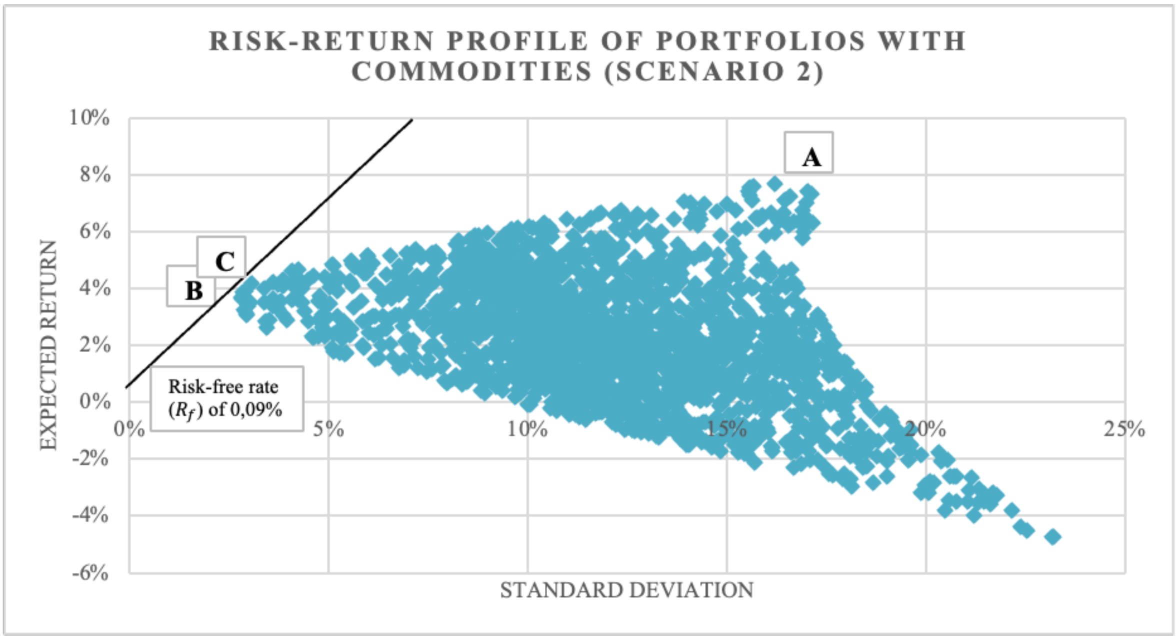
A variety of investment choices, all differing from one another by the asset weight in the portfolio, is shown in Figure 19.

Unlike *Scenario 1*, these portfolios consist of three different asset classes. According to Fabozzi and Grant (2001), if a portfolio of more than two assets is created, a feasible set of portfolios is no longer a curve, as it takes up an entire area, as evidenced by Figure 10.

In Figure 19, three points of interest were identified: the point with the maximum expected return (point A), the point with the minimum standard deviation (point B) and the point with the maximum Sharpe ratio (point C). The risk-return profile od *Scenario 2*, showing the points A to C, is presented in Figure 20.

Figure 20

Risk-return profile of portfolios with commodities (Scenario 2) showing the portfolios A to C (02.01.2004 to 01.01.2021)



Note. Source: Own calculation.

Points A to C have the following composition of indices (see Table 12):

Table 12

Weights, expected returns and standard deviations of points A to C from Figure 20

Point	Description	Weights MSCI World Index - Total Return (USD)	Weights FTSE World Government Bond Index - Total Return (USD)	Weights S&P GSCI Commodity Index - Total Return (USD)	Expected return	Standard Deviation
A	Max. Expected Return	91.30%	8.70%	0.00%	7.70%	16.19%
B	Min. Standard Deviation	6.25%	90.63%	3.13%	3.63%	2.81%
C	Max. Sharpe Ratio	12.90%	87.10%	0.00%	4.19%	3.08%

Note. Source: Own calculation.

Since the randomly assumed weights did not include the option of investing 100% in the stock index, the MSCI World Index - Total Return, a separate calculation was made in order to calculate the expected return and standard deviation of a portfolio composed solely of the MSCI World Index - Total Return. The maximum expected return of this portfolio was 8.09% with a volatility of 17.80% (just like in *Scenario 1*, where the option of investing solely in the stock index was included in the randomly assumed weights), making this portfolio the one with the highest expected return. However, since the randomly assumed weights did not include this option in the calculations, Table 12 and all further conclusions do not contain the option of investing 100% in the stock index, the MSCI World Index - Total Return at all.

The maximum expected return among the portfolios that were randomly chosen belongs to the portfolio with 91.30% invested in the stock index, the MSCI World Index - Total Return and 8.70% invested in the bond index, the FTSE World Government Bond Index - Total Return. Within two thousand randomly generated portfolios in *Scenario 2*, where the portfolio consists of stocks, bonds and commodity indices, the portfolio with the minimum standard deviation was the one in which the majority invested was in the bond index, the FTSE World Government Bond Index - Total Return, with 90.63%, while 6.25% were invested in the stock index, the MSCI World Index - Total Return and 3.13% in the commodity index, the S&P GSCI Commodity Index - Total Return. As in Scenario 1, the portfolio with the highest Sharpe ratio was analysed as well and the majority of the portfolio with the highest Sharpe ratio is invested in the bond index, i.e. the FTSE World Government Bond Index - Total Return, with 87.10%, 12.90% in the stock index, i.e. MSCI World Index - Total Return and 0% in the commodity index, the S&P GSCI Commodity Index - Total Return. This is in line with the findings from Sub-chapter 5.3.1, since the commodity index, the S&P GSCI Commodity Index - Total Return had a negative average annual rate of return and a very high volatility, influencing the way the weights are distributed between the assets within the portfolio (see Table 12).

Value at Risk

In *Scenario 2*, the minimum VaR amounts to 27,448.32 USD, while the maximum VaR amounts to 586,963.27 USD. The portfolio composition with the minimum VaR is as follows: MSCI World Index - Total Return (representing stocks) weighting 8.57%, FTSE World Government Bond (representing bonds) weighting 90.00% and S&P GSCI Commodity Index - Total Return (representing commodities) weighting 1.43%. The minimum VaR indicates that this portfolio composition might be more favorable to invest in, due to the lowest amount of money one might lose when investing in it (Hull, 2014).

On the other hand, the portfolio composition with the maximum VaR (MSCI World Index - Total Return weighting 0.00%, FTSE World Government Bond weighting 1.18% and S&P GSCI Commodity Index - Total Return weighting 98.82%.) is the least favorable investment option, since it may result in the highest amount of money being lost (Hull, 2014).

Sharpe Ratio

The highest Sharpe ratio and therefore the superior investment option in this Scenario amounts to 1.333932322, boasting the following portfolio composition: MSCI World Index - Total Return (representing stocks) weighting 12.90%, FTSE World Government Bond (representing bonds) weighting 87.10% and S&P GSCI Commodity Index - Total Return (representing commodities) weighting 0.00%. On the other hand, the portfolio composed of 0.00% of the MSCI World Index - Total Return, 1.18% of the FTSE World Government Bond and 98.82% of the S&P GSCI Commodity Index - Total Return has the lowest Sharpe ratio of -0.208652201 and is therefore the least favorable investment option (Anuwoje and Togborlo, 2019).

The portfolio that should be chosen in the *tangency portfolio* (i.e. market portfolio), is the one where the steepest line, called the *Capital Market Line* (CML), touches the efficient frontier, as outlined in Sub-chapter 3.1.4 (Schulmerich, 2013). Therefore, the aim is to maximize the Sharpe ratio, being that the Sharpe ratio represents the excess portfolio return over the risk-free rate (R_f), divided by portfolio volatility (Schulmerich, 2013).

As outlined above, the maximum Sharpe ratio in this empirical analysis amounts to 1.333932322 and is presented as point C (the portfolio consisting of 12.90% of the MSCI World Index - Total Return, 87.10% of the FTSE World Government Bond and 0.00% of the S&P GSCI Commodity Index - Total Return) in Figure 20, making it the tangency portfolio (i.e. market portfolio or optimal portfolio) (Schulmerich, 2013).

5.5.5. Portfolio optimization (Scenario 3)

Similar to *Scenario 2*, this scenario relies on a portfolio consisting of three asset classes. The portfolio in this instance is composed of stocks (with stock price movements represented by the MSCI World Index - Total Return), bonds (with bond price movements represented by the FTSE World Government Bond Index – Total Return) and commodities (with commodity price movements represented by the Rogers International Commodity Index - Total Return RICI).

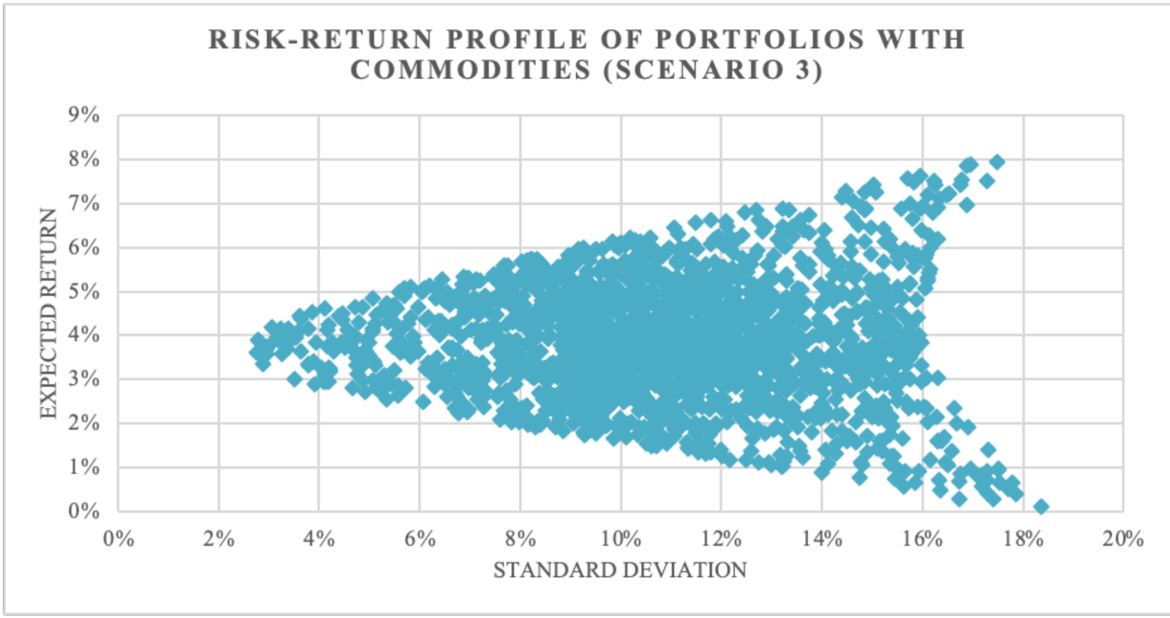
In order to calculate the portfolio optimization in *Scenario 3*, two thousand random asset weights are generated, as in previous scenarios, with the sum of each portfolio equaling one. The average rate of return from Sub-chapter 5.3.1 for the stock, bond and commodity indices (MSCI World Index - Total Return, FTSE World Government Bond and Rogers International Commodity Index - Total Return RICI) is used to calculate the expected return of each asset in the portfolio. The variance calculation (and therefore also the standard deviation) per assumed asset weight is calculated by using the previously generated covariance matrix from Sub-chapter 5.3.2. The same goes for the Sharpe ratio and Value at Risk (VaR).

Risk-Return Profile

A risk-return profile for Scenario 3 shows a variety of investment choices, all differing from one another by the asset weight in the portfolio (see Figure 21). These portfolios are composed of stocks (represented by the MSCI World Index - Total Return), bonds (represented by the FTSE World Government Bond Index -cTotal Return) and commodities (represented by the Rogers International Commodity Index - Total Return RICI).

Figure 21

Risk-return profile of portfolios with commodities in Scenario 3 (02.01.2004 to 01.01.2021)



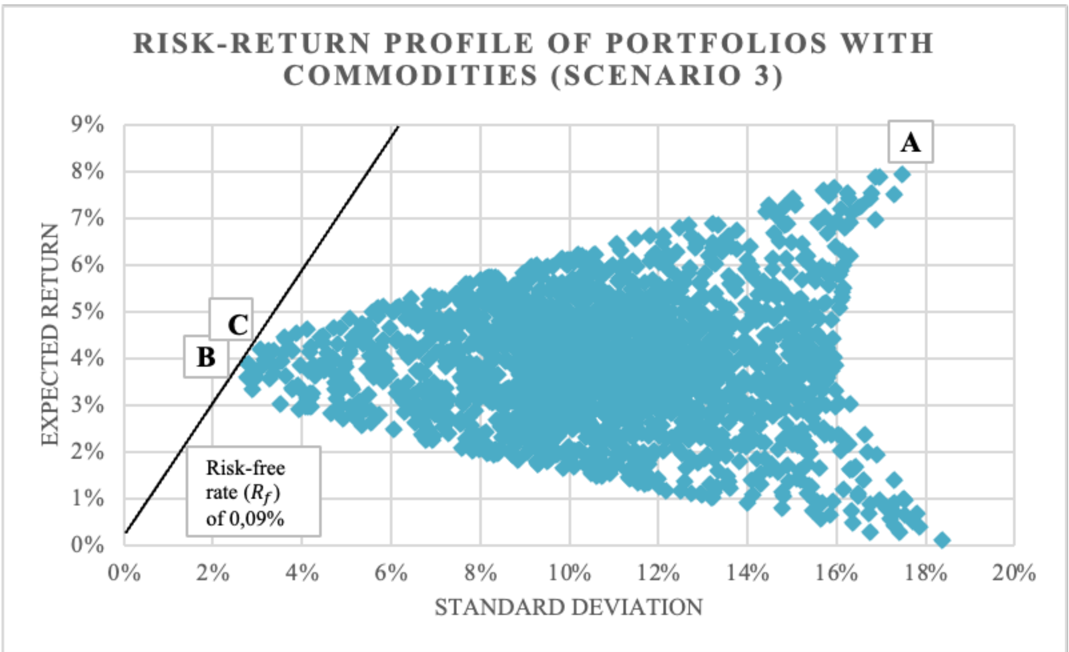
Note. Source: Own calculation.

Like in *Scenario 2*, these portfolios consist of three different asset classes and the feasible set of portfolios is therefore no longer a curve, as it takes up an entire area instead (Fabozzi and Grant, 2001), as evidenced by Figure 21.

In Figure 21, three points of interest were identified: the point with the maximum expected return (point A), the point with the minimum standard deviation (point B) and the point with the maximum Sharpe ratio (point C). The risk-return profile od *Scenario 3*, showing the points A to C, is presented in Figure 22.

Figure 22

Risk-return profile of portfolios with commodities (Scenario 3) showing the portfolios A to C (02.01.2004 to 01.01.2021)



Note. Source: Own calculation.

The composition of portfolios A to C, with their expected returns and standard deviations, is shown in Table 13:

Table 13

Weights, expected returns and standard deviations of points A to *C from Figure 22*

Point	Description	Weights MSCI World Index - Total Return (USD)	Weights FTSE World Government Bond Index - Total Return (USD)	Weights Rogers International Commodity Index - Total Return (USD)	Expected return	Standard Deviation
A	**Max. Expected Return**	97.53%	1.23%	1.23%	7.93%	17.47%
B	**Min. Standard Deviation**	3.33%	92.22%	4.44%	3.60%	2.77%
C	**Max. Sharpe Ratio**	6.86%	92.16%	0.98%	3.89%	2.79%

Note. Source: Own calculation.

Since the randomly assumed weights did not include the option of investing 100% in the stock index, the MSCI World Index - Total Return, a separate calculation was made in order to calculate the expected return and standard deviation of a portfolio composed solely of the MSCI World Index - Total Return. The maximum expected return of this portfolio was 8.09% with a volatility of 17.80% (just like in *Scenario 1*, where the option of investing solely in the stock index was included in the randomly assumed weights), making this portfolio the one with the highest expected return. However, since the randomly assumed weights did not include this option in the calculations, Table 13 and all further conclusions do not contain the option of investing 100% in the stock index, the MSCI World Index - Total Return at all.

The portfolio with the highest expected return among the portfolios that were randomly chosen is the portfolio in which 97.53% is invested in the stock index, the MSCI World Index - Total Return, 1.23% in the bond index, the FTSE World Government Bond Index - Total Return and 1.23% in the commodity index, the Rogers International Commodity Index - Total Return. However, the portfolio with the minimum standard deviation was once again the one in which the majority invested was in the bond index, the FTSE World Government Bond Index - Total Return, with 92.22%, while 3.33% were invested in the stock index, the MSCI World Index - Total Return and 4.44% in the commodity index, the Rogers International Commodity Index - Total Return. As in *Scenarios 1* and *2*, the portfolio with the highest Sharpe ratio was analysed as well and once again the majority of the portfolio with the highest Sharpe ratio is invested in the bond index, i.e. the FTSE World Government

Bond Index - Total Return, with 92.16%, 6.86% in the stock index, the MSCI World Index - Total Return and 0.98% in the commodity index, the Rogers International Commodity Index - Total Return. This is in line with the findings from Sub-chapter 5.3.1, since the commodity index, the Rogers International Commodity Index - Total Return, had a negative average annual rate of return and a very high volatility, similar to the S&P GSCI Commodity Index - Total Return from *Scenario 2*, influencing the way the weights are distributed between the assets within the portfolio (see Table 13).

Value at Risk

In *Scenario 3*, the minimum VaR amounts to 26,049.54 USD, while the maximum VaR amounts to 426,342.19 USD. The portfolio composition with the minimum VaR is as follows: MSCI World Index - Total Return (representing stocks) weighting 6.86%, FTSE World Government Bond Index – Total Return (representing bonds) weighting 92.16% and Rogers International Commodity Index - Total Return RICI (representing commodities) weighting 0.98%. On the other hand, the portfolio composition with the maximum VaR (MSCI World Index - Total Return weighting 2.04%, FTSE World Government Bond weighting 1.02% and Rogers International Commodity Index - Total Return RICI weighting 96.94%.) is the least favorable investment option, since it may result in the highest amount of money being lost. The minimum VaR indicates that this portfolio composition might be more favorable to invest in, due to the lowest amount of money one might lose when investing in it (Hull, 2014).

Sharpe Ratio

The highest Sharpe ratio in Scenario 3 amounts to a Sharpe ratio of 1.360619955, with the following portfolio composition: MSCI World Index - Total Return (representing stocks) weighting 6.86%, FTSE World Government Bond Index – Total Return (representing bonds) weighting 92.16% and Rogers International Commodity Index - Total Return RICI (representing commodities) weighting 0.98%. On the other hand, the least favorable investment option in *Scenario 3* is the portfolio consisting of 2.04% of the MSCI World Index - Total Return, 1.02% of the FTSE World Government Bond Index – Total Return and 96.94% of the Rogers International Commodity Index - Total Return RICI, with the lowest Sharpe ratio of 0.000659674.

Since the aim is to maximize the Sharpe ratio, the tangency portfolio (i.e. market portfolio or optimal portfolio) in this scenario is the one with the maximum Sharpe ratio, found in point C.

5.6. Five-year empirical analysis

A shorter, empirical analysis of the last five years, from 01.01.2016 to 01.01.2021, was conducted as well, in order to evaluate whether there are major differences in (1) the risk-return profiles of each total return index, (2) correlation and covariance matrices and (3) the scenarios of portfolio optimization over the last five years when compared with the original seventeen-year time period.

In this empirical analysis, the prices of the same four total return indices (the MSCI World Index - Total Return for stock price movements; the FTSE World Government Bond Index - Total Return for bond price movements; the S&P GSCI Commodity Index - Total Return for commodity price movements and the Rogers International Commodity Index - Total Return (RICI) Index for commodity price movements), all denominated in the same currency (USD), were chosen as well. However, weekly index prices from 01.01.2016 to 01.01.2021 were used instead – for a further risk and return analysis and portfolio optimizations.

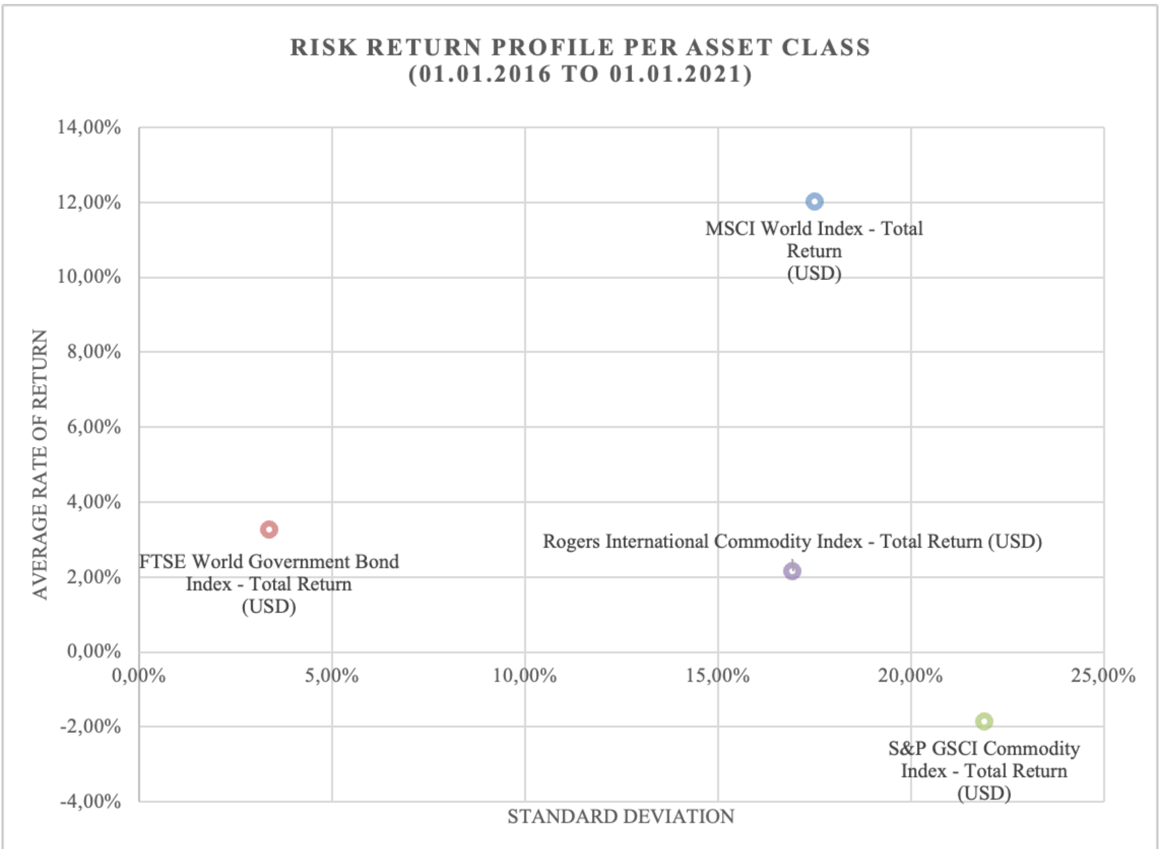
In the risk and return part of this five-year empirical analysis, the abovementioned four indices were compared in terms of their return and volatility in the period from 01.01.2016 to 01.01.2021 (five years in total). Additionally, the correlation and covariance between these three asset classes were calculated for this shorter time span as well. Different portfolio optimization calculations (three different scenarios) were calculated and analysed, however again by using data from a shorter (five-year) time span. In order to calculate the portfolio optimization, the same two thousand random asset weights generated earlier were used in this five-year empirical analysis, with the assumption that the sum of each portfolio equals one.

Risk and return profile

In this five-year empirical analysis, weekly and annual rates of return of each individual index, as well as the standard deviation, were calculated based on weekly market prices during the time period of five years (01.01.2016 to 01.01.2021). The annual rate of return and the standard deviation both help build a risk-return profile of the asset classes analysed, as presented in Figure 23 and Table 14.

Figure 23

Risk-return profile per asset class (01.01.2016 to 01.01.2021)



Note. Source: Own calculation.

Table 14

Risk-return profile of asset classes (01.01.2016 to 01.01.2021)

	MSCI World Index - Total Return (USD)	FTSE World Government Bond Index - Total Return (USD)	S&P GSCI Commodity Index - Total Return (USD)	Rogers International Commodity Index - Total Return (USD)
Average annual rate of return	12.01%	3.27%	-1.86%	2.17%
Annual standard deviation	17.51%	3.37%	21.91%	16.94%

Note. Source: Own calculation.

It can be concluded that the MSCI World Index - Total Return, a stock index, has the highest average annual rate of return of 12.01% over the period of time from 01.01.2016 to 01.01.2021, with a high volatility of 17.51%. However, the highest volatility of 21.91% is associated with the commodity index, the S&P GSCI Commodity Index - Total Return. The Rogers International Commodity Index - Total Return records a lower, but still high volatility of 16.94% and an average annual rate of return of 2.17%. The other commodity index used in this empirical analysis, the S&P GSCI Commodity Index - Total Return, records a negative average annual rate of return of -1.86%, with the highest volatility among the analysed indices, as already mentioned above. The FTSE World Government Bond Index - Total Return, a bond index, recorded the lowest volatility among the analysed indices, with 3.37%, while generating a higher (and positive) average rate of return than both analysed commodity indices, with 3.27%.

Correlation and covariance

The correlation and covariance matrices were observed over the course of the last five years of the originally analysed seventeen-year timeline (from 01.01.2016 to 01.01.2021) in all of the three scenarios calculated.

Scenario 1

In *Scenario 1* for the period from 01.01.2016 to 01.01.2021, the portfolio composed exclusively of stocks and bonds, represented by the MSCI World Index - Total Return (for stocks) and the FTSE World Government Bond Index - Total Return (for bonds), shows a negative low correlation and a negative covariance between the stock and the bond index, i.e. the assets are changing in opposite ways, as shown in Table 15 and Table 16 and as outlined by Fabozzi and Grant (2001) and Das (2003).

Table 15

Correlation matrix for the MSCI World Index - Total Return and the FTSE World Government Bond Index - Total Return (01.01.2016 to 01.01.2021)

Correlation Matrix	MSCI World Index - Total Return (USD)	FTSE World Government Bond Index - Total Return (USD)
MSCI World Index - Total Return (USD)	1	-0.078871201
FTSE World Government Bond Index - Total Return (USD)	-0.078871201	1

Note. Source: Own calculation.

Table 16

Covariance matrix for the MSCI World Index - Total Return and the FTSE World Government Bond Index - Total Return (01.01.2016 to 01.01.2021)

Covariance Matrix	MSCI World Index - Total Return (USD)	FTSE World Government Bond Index - Total Return (USD)
MSCI World Index - Total Return (USD)	0.000589538	-0.00000895
FTSE World Government Bond Index - Total Return (USD)	-0.00000895	0.00002185

Note. Source: Own calculation.

Scenario 2

In *Scenario 2* for the period from 01.01.2016 to 01.01.2021, the S&P GSCI Commodity Index - Total Return is added to the portfolio, so that the portfolio consists of stocks, bonds and commodities – and is represented by the MSCI World Index - Total Return (for stocks), the FTSE World Government Bond Index - Total Return (for bonds) and the S&P GSCI Commodity Index - Total Return (for commodities).

The commodity index analysed in this scenario (S&P GSCI Commodity Index - Total Return) shows a low negative correlation and a negative covariance with the FTSE World Government Bond Index - Total Return (bonds) and a high positive correlation and a positive covariance with the MSCI World Index - Total Return (stocks), indicating that these two assets, the MSCI World Index - Total Return and the S&P GSCI Commodity Index - Total Return, change in the same way while the S&P GSCI Commodity Index - Total Return and the FTSE World Government Bond Index - Total Return (bonds) change in the opposite way, as shown in Table 17 and Table 18 and outlined by Fabozzi and Grant (2001) and Das (2003).

Table 17

Correlation matrix for the MSCI World Index - Total Return, the FTSE World Government Bond Index - Total Return and the S&P GSCI Commodity Index - Total Return (01.01.2016 to 01.01.2021)

Correlation Matrix	MSCI World Index - Total Return (USD)	FTSE World Government Bond Index - Total Return (USD)	S&P GSCI Commodity Index - Total Return (USD)
MSCI World Index - Total Return (USD)	1	-0.0788712009	0.5549663544
FTSE World Government Bond Index - Total Return (USD)	-0.0788712009	1	-0.1104063056
S&P GSCI Commodity Index - Total Return (USD)	0.5549663544	-0.1104063056	1

Note. Source: Own calculation.

Table 18

Covariance matrix for the MSCI World Index - Total Return, the FTSE World Government Bond Index - Total Return and the S&P GSCI Commodity Index - Total Return (01.01.2016 to 01.01.2021)

Covariance Matrix	MSCI World Index - Total Return (USD)	FTSE World Government Bond Index - Total Return (USD)	S&P GSCI Commodity Index - Total Return (USD)
MSCI World Index - Total Return (USD)	0.000589538	-0.000008952	0.000409365
FTSE World Government Bond Index - Total Return (USD)	-0.000008952	0.000021853	-0.000015680
S&P GSCI Commodity Index - Total Return (USD)	0.000409365	-0.000015680	0.000922947

Note. Source: Own calculation.

Scenario 3

In *Scenario 3* for the period from 01.01.2016 to 01.01.2021, the Rogers International Commodity Index - Total Return (RICI) is added to the portfolio from *Scenario 1*, so that the portfolio consists of stocks, bonds and commodities – and is represented by the MSCI World Index - Total Return (for stocks), the FTSE World Government Bond Index - Total Return (for bonds) and the Rogers International Commodity Index - Total Return (RICI) (for commodities). The Rogers International Commodity Index - Total Return shows a low negative correlation and a negative covariance with the FTSE World Government Bond Index - Total Return (bonds) and a high positive correlation and a positive covariance with the MSCI World Index - Total Return (stocks), indicating that the MSCI World Index - Total Return and the Rogers International Commodity Index - Total Return change in the same way and the FTSE World Government Bond Index - Total Return and the Rogers International Commodity Index - Total Return change in opposite ways, as outlined by Fabozzi and Grant (2001) and Das (2003) and shown in Table 19 and Table 20.

Table 19

Correlation matrix for the MSCI World Index - Total Return, the FTSE World Government Bond Index - Total Return and the Rogers International Commodity Index - Total Return RICI (01.01.2016 to 01.01.2021)

Correlation Matrix	MSCI World Index - Total Return (USD)	FTSE World Government Bond Index - Total Return (USD)	Rogers International Commodity Index - Total Return (USD)
MSCI World Index - Total Return (USD)	1	-0.078871201	0.602989756
FTSE World Government Bond Index - Total Return (USD)	-0.078871201	1	-0.11222465
Rogers International Commodity Total Return Index (USD)	0.602989756	-0.11222465	1

Note. Source: Own calculation.

Table 20

Covariance matrix for the MSCI World Index - Total Return, the FTSE World Government Bond Index - Total Return and the Rogers International Commodity Index - Total Return RICI (01.01.2016 to 01.01.2021)

Covariance Matrix	MSCI World Index - Total Return (USD)	FTSE World Government Bond Index - Total Return (USD)	Rogers International Commodity Index - Total Return (USD)
MSCI World Index - Total Return (USD)	0.000589538	-0.000008952	0.000343897
FTSE World Government Bond Index - Total Return (USD)	-0.000008952	0.000021853	-0.000012323
Rogers International Commodity Total Return Index (USD)	0.000343897	-0.000012323	0.000551729

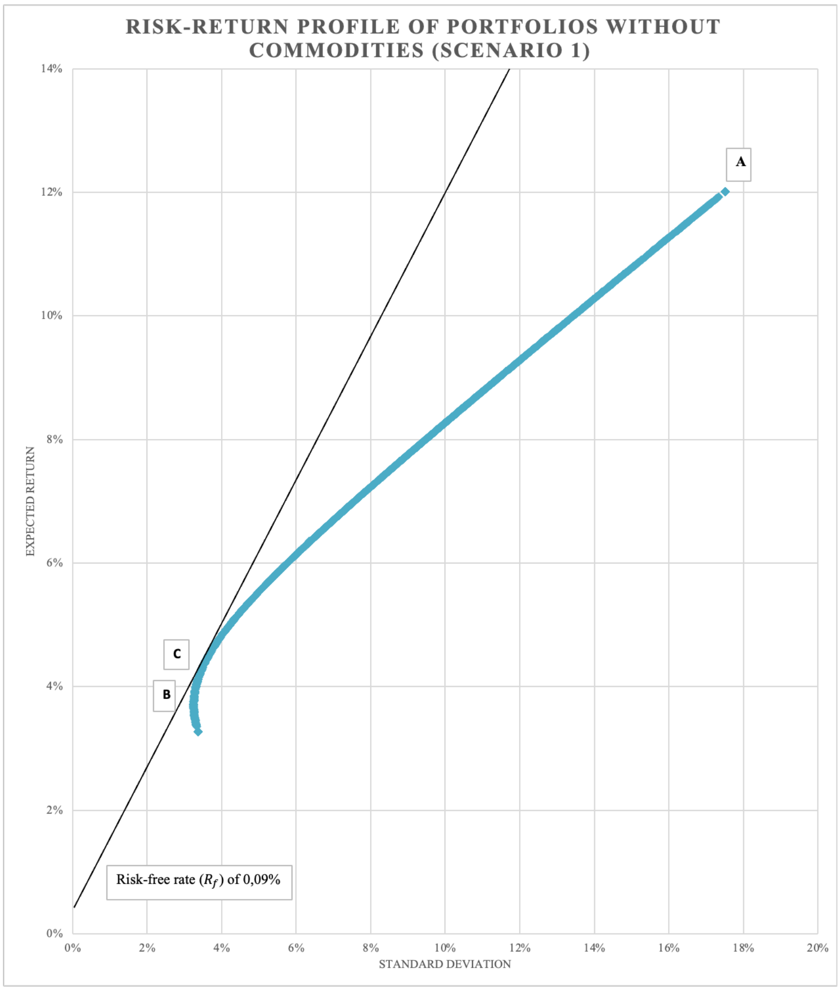
Note. Source: Own calculation.

Portfolio optimization (Scenario 1)

After calculating both the expected return and the standard deviation of each assumed portfolio for the five-year analysis (01.01.2016 to 01.01.2021), as described in Sub-chapter 5.5.1 and in the same manner as calculated in the portfolio optimization for *Scenario 1* for the longer timeline (see Sub-chapter 5.5.3), a risk-return profile of portfolios composed exclusively of stocks (represented by weekly returns of the MSCI World Index - Total Return) and bonds (represented by weekly returns of the FTSE World Government Bond) can be plotted and is presented in Figure 24.

Figure 24

Risk-return profile of portfolios without commodities (Scenario 1) showing the portfolios A to C (01.01.2016 to 01.01.2021)



Note. Source: Own calculation.

Three points of interest were identified: the point with the maximum expected return (point A), the point with the minimum standard deviation (point B) and the point with the maximum Sharpe ratio (point C), due to its connection with superior investment opportunities, as per Anuwoje and Togborlo (2019). Portfolios A to C, with their expected returns and standard deviations, have the following composition of indices (see Table 21):

Table 21

Weights, expected returns and standard deviations of points A to C from Figure 24

Point	Description	Weights MSCI World Index - Total Return (USD)	Weights FTSE World Government Bond Index - Total Return (USD)	Expected return	Standard Deviation
A	**Max. Expected Return**	100.00%	0.00%	12.01%	17.51%
B	**Min. Standard Deviation**	4.81%	95.19%	3.69%	3.25%
C	**Max. Sharpe Ratio**	12.73%	87.27%	4.39%	3.55%

Note. Source: Own calculation.

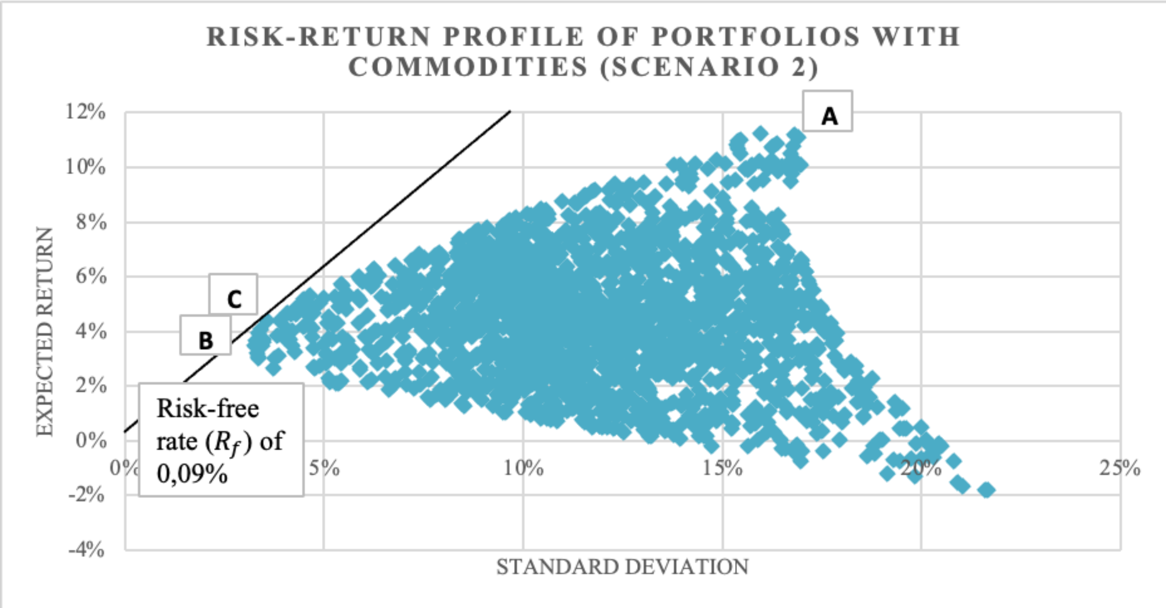
Of two thousand randomly generated portfolios in *Scenario 1* for the period from 01.01.2016 to 01.01.2021, where the portfolio consists of stock and bond indices, the portfolio with 100% invested in the stock index, the MSCI World Index - Total Return and 0% invested in the bond index, the FTSE World Government Bond Index - Total Return, had the maximum expected return of 12.01%. The portfolio with the minimum standard deviation was the one in which the majority invested was in the bond index, the FTSE World Government Bond Index - Total Return, with 95.19%, while 4.81% were invested in the stock index, the MSCI World Index - Total Return. Since the highest Sharpe ratio is linked to superior investment opportunities, this portfolio was analysed as well (Anuwoje and Togborlo, 2019). The majority of the portfolio with the highest Sharpe ratio is invested in the bond index, i.e. the FTSE World Government Bond Index - Total Return, with 87.27%, and 12.73% in the stock index, the MSCI World Index - Total Return. These findings confirm those from the risk return profile analysed earlier on, since the MSCI World Index - Total Return, a stock index, had a higher average rate of return than the FTSE World Government Bond Index - Total Return, a bond index, which recorded a much lower volatility than the stock index.

Portfolio optimization (Scenario 2)

As in *Scenario 1*, after calculating the expected return per random assumed portfolio and the variance (and therefore also the standard deviation) of each portfolio by using the previously generated covariance matrix from Sub-chapter 5.6, a risk-return profile for the five-year analysis (01.01.2016 to 01.01.2021) was composed. This risk-return profile for *Scenario 2* for the five-year analysis consisting of stocks (represented by the MSCI World Index - Total Return), bonds (represented by the FTSE World Government Bond Index – Total Return) and commodities (represented by the S&P GSCI Commodity Index - Total Return) is shown in Figure 25.

Figure 25

Risk-return profile of portfolios with commodities (Scenario 2) showing the portfolios A to C (01.01.2016 to 01.01.2021)



Note. Source: Own calculation.

Unlike *Scenario 1*, since these portfolios consist of three different asset classes, the feasible set of portfolios is no longer a curve, as it takes up an entire area, as evidenced by Figure 25 (Fabozzi and Grant, 2001). In Figure 25, three points of interest were identified: the point with the maximum expected return (point A), the point with the minimum standard deviation (point B) and the point with the maximum Sharpe ratio (point C), with the following composition of indices (see Table 22):

Table 22

Weights, expected returns and standard deviations of points A to C from Figure 25

Point	Description	Weights MSCI World Index - Total Return (USD)	Weights FTSE World Government Bond Index - Total Return (USD)	Weights S&P GSCI Commodity Index - Total Return (USD)	Expected return	Standard Deviation
A	Max. Expected Return	91.30%	8.70%	0.00%	11.25%	15.97%
B	Min. Standard Deviation	3.06%	95.92%	1.02%	3.49%	3.24%
C	Max. Sharpe Ratio	12.90%	87.10%	0.00%	4.40%	3.56%

Note. Source: Own calculation.

Since the randomly assumed weights did not include the option of investing 100% in the stock index, the MSCI World Index - Total Return, a separate calculation was made in order to calculate the expected return and standard deviation of a portfolio composed solely of the MSCI World Index - Total Return. The maximum expected return of this portfolio was 12.01% with a volatility of 17.51% (just like in case of *Scenario 1*, where the option of investing solely in the stock index was included in the randomly assumed weights), making this portfolio the one with the highest expected return. However, since the randomly assumed weights did not include this option in the calculations, Table 22 and all further conclusions do not contain the option of investing 100% in the stock index, the MSCI World Index - Total Return at all.

The maximum expected return among the portfolios that were randomly chosen belongs to the portfolio with 91.30% invested in the stock index, the MSCI World Index - Total Return and 8.70% invested in the bond index, the FTSE World Government Bond Index - Total Return. Within two thousand randomly generated portfolios in *Scenario 2*, where the portfolio consists of stocks, bonds and commodity indices, the portfolio with the minimum standard deviation was the one in which the majority invested was in the bond index, the FTSE World Government Bond Index - Total Return, with 95.92%, while 3.06% were invested in the stock index, the MSCI World Index - Total Return and 1.02% in the commodity index, the S&P GSCI Commodity Index - Total Return. Like in *Scenario 1*, the portfolio with the highest Sharpe ratio was analysed as well and the majority of the portfolio with the highest Sharpe ratio is invested in the bond index, i.e. the FTSE World Government Bond Index - Total Return, with 87.10%, 12.90% in the stock index, i.e. MSCI World Index - Total Return and 0% in the commodity index, the S&P GSCI Commodity Index - Total

Return. This is in line with the findings of the risk return profile, since the commodity index, the S&P GSCI Commodity Index - Total Return had a negative average annual rate of return and the highest volatility, influencing the way the weights are distributed between the assets within the portfolio (see Table 22).

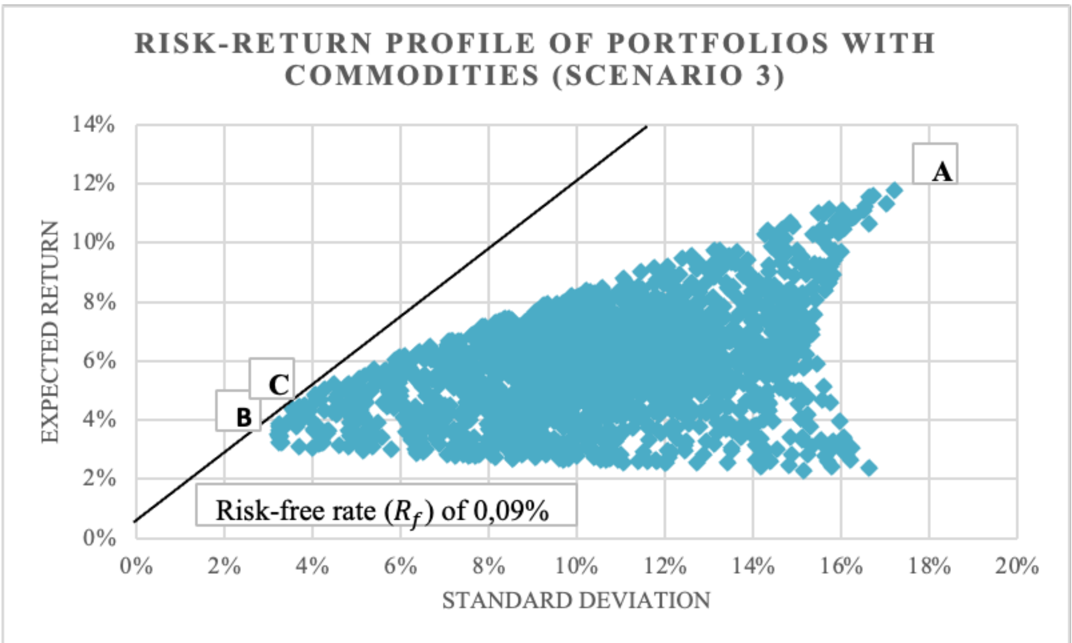
Portfolio optimization (Scenario 3)

As in *Scenario 2*, this scenario relies on a portfolio consisting of three asset classes: stocks (with stock price movements represented by the MSCI World Index - Total Return), bonds (with bond price movements represented by the FTSE World Government Bond Index – Total Return) and commodities (with commodity price movements represented by the Rogers International Commodity Index - Total Return RICI).

In a similar manner as in *Scenario 2*, after calculating the expected return and variance per randomly assumed portfolio, a risk-return profile for the five-year analysis (01.01.2016 to 01.01.2021) was composed. This risk-return profile for *Scenario 3* for the five-year analysis is shown in Figure 26.

Figure 26

Risk-return profile of portfolios with commodities in Scenario 3 (01.01.2016 to 01.01.2021)



Note. Source: Own calculation.

In Figure 26, three points of interest were identified: the point with the maximum expected return (point A), the point with the minimum standard deviation (point B) and the point with the maximum Sharpe ratio (point C), with the following composition of indices (see Table 23):

Table 23

Weights, expected returns and standard deviations of points A to C from Figure 26

Point	Description	Weights MSCI World Index - Total Return (USD)	Weights FTSE World Government Bond Index - Total Return (USD)	Weights Rogers International Commodity Index - Total Return (USD)	Expected return	Standard Deviation
A	Max. Expected Return	97.53%	1.23%	1.23%	11.79%	17.20%
B	Min. Standard Deviation	3.33%	92.22%	4.44%	3.52%	3.21%
C	Max. Sharpe Ratio	12.73%	87.27%	0.00%	4.39%	3.55%

Note. Source: Own calculation.

Since the randomly assumed weights did not include the option of investing 100% in the stock index, the MSCI World Index - Total Return, a separate calculation was made in order to calculate the expected return and standard deviation of a portfolio composed solely of the MSCI World Index - Total Return. The maximum expected return of this portfolio was 12.01% with a volatility of 17.51% (just like in case of *Scenario 1*, where the option of investing solely in the stock index was included in the randomly assumed weights), making this portfolio the one with the highest expected return. However, since the randomly assumed weights did not include this option in the calculations, Table 23 and all further conclusions do not contain the option of investing 100% in the stock index, the MSCI World Index - Total Return at all.

The highest expected return among the portfolios that were randomly chosen is assigned to the portfolio in which 97.53% is invested in the stock index, the MSCI World Index - Total Return, 1.23% in the bond index, the FTSE World Government Bond Index - Total Return and 1.23% in the commodity index, the Rogers International Commodity Index - Total Return. However, the portfolio with the minimum standard deviation was once again the one in which the majority invested was in the bond index, the FTSE World Government Bond Index - Total Return, with 92.22%, while 3.33% were invested in the stock index, the MSCI World Index - Total Return and 4.44% in the commodity index, the Rogers International Commodity Index - Total Return. As in *Scenarios 1* and *2*, the portfolio with the highest Sharpe ratio was analysed as well and once again the majority of the portfolio with the highest Sharpe ratio is invested in the bond index, i.e. the FTSE World Government Bond Index - Total Return, with 87.27%, 12.73% in the stock index, the MSCI World Index - Total Return and 0.00% in the commodity index, the Rogers International Commodity Index -

Total Return. This is in line with the findings from the risk-return profile for the five-year empirical analysis, since the commodity index, the Rogers International Commodity Index - Total Return, had a low average annual rate of return and a very high volatility, not too dissimilar to the negative annual rate of return of the S&P GSCI Commodity Index - Total Return from *Scenario 2*, impacting the way the weights are distributed between the assets within the portfolio.

5.7. Analysis of results

The hypothesis of this thesis – that global investments perform better (have a better risk-to-return ratio) with commodities, than without them and that by including commodities in the portfolio it enhances the portfolio performance, due to positive effects of diversification, which is why global investors' portfolio should include commodities in their respective portfolios– is put to the test with an empirical analysis.

This search for a definitive answer to the question of whether or not adding commodities to a portfolio enhances its quality in terms of return and volatility resulted in the following set of conclusion:

- When compared to other asset classes observed as part of the seventeen-year empirical analysis (stocks and bonds), the annual rates of return of the two commodity indices were very low (negative), with very high volatility (see Table 1), making them, when observed on their own merit, a less attractive investment option. A similar conclusion can be drawn when taking a closer look at the risk-return profiles of the five-year empirical analysis (01.01.2016 to 01.01.2021), the difference being that the annual rate of return of the Rogers International Commodity Index - Total Return (USD) is low, but positive, as opposed to the risk-return analysis of the seventeen-year empirical analysis, where this commodity index had a negative annual rate of return.

- One of the main advantages of commodities is their very low correlation to equity and combinations of equity and bonds (Kayser, Paris and Ross, 2011). However, in the seventeen-year empirical analysis, a low negative correlation of commodity indices was discovered only in the case of the bond index, the FTSE World Government Bond Index - Total Return. On the other hand, high positive correlations with the stock index, the MSCI World Index - Total Return, were recorded with both commodity indices, the Rogers International Commodity Index - Total Return and the S&P GSCI Commodity Index - Total Return. The same conclusion can be drawn when taking a closer look at the correlation and covariance matrices that are a part of the five-year empirical analysis (01.01.2016 to 01.01.2021).

- It can be deduced from the histograms of the seventeen-year empirical analysis (see Sub-chapter 5.4.1) that the normal standard distribution and the Kernel density of the empirical data used did not overlap in all four indices analysed and that data normality can therefore not be assumed in these cases. A similar image was seen when analysing the Q-Q Plots, since the observed data and the expected data did not overlap. Additionally, the skewness and kurtosis analysis indicated that a deviation from normal distribution exists, due to the indices being negatively skewed with a kurtosis differing from 3. Based on the histograms, the Q-Q Plots, the skewness and kurtosis analysis and the Kolmogorov-Smirnov and the Shapiro-Wilk test conducted, data normality could not be assumed for this data set, as per interpretation of Mishra et al. (2019) and Anuwoje and Togborlo (2019).

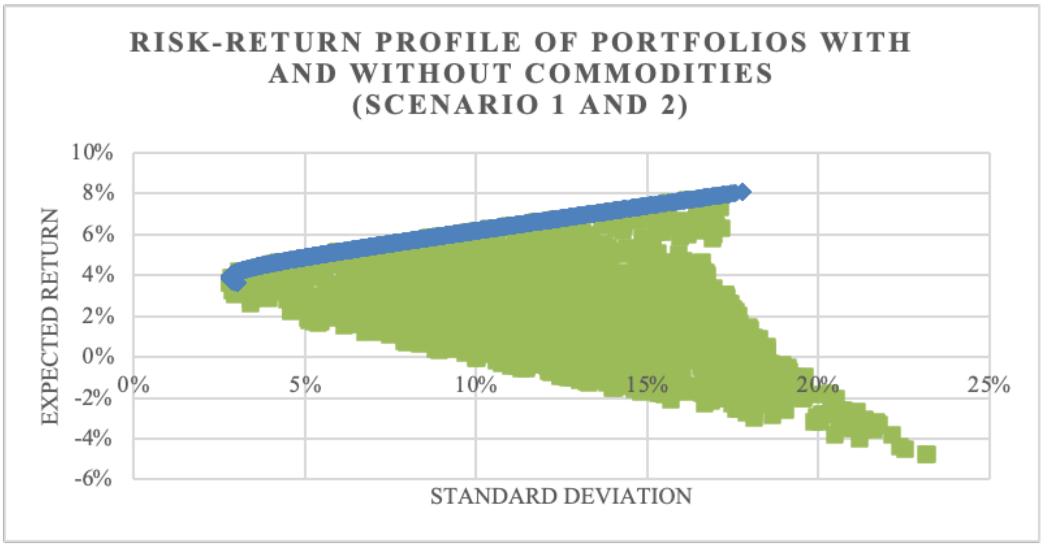
- In *Scenario 2* of the seventeen-year empirical analysis, the maximum Sharpe ratio is found in a portfolio with a 0.00% weight of a commodity index (S&P GSCI Commodity Index - Total Return). Therefore, the tangency portfolio (i.e. market or optimal portfolio) that should be selected as the most favorable portfolio in *Scenario 2* does not include commodities at all. In *Scenario 3* of the seventeen-year empirical analysis, the portfolio with the highest Sharpe ratio is the one where commodities (Rogers International Commodity Index - Total Return RICI) account for only 0.98% of the portfolio. These results lead to the conclusion that commodities being included in *Scenario 2* did not contribute to a better risk-return ratio or to an overall better portfolio performance at all. However, in *Scenario 3* of the seventeen-year empirical analysis, by investing a smaller part of the portfolio in the Rogers International Commodity Index - Total Return RICI, due to its slightly higher average annual rate of return and lower volatility compared to the other commodity index (S&P GSCI Commodity Index - Total Return) used, higher Sharpe ratio and a minimal VaR, i.e. a better risk-return ratio, could be achieved.

- On the other hand, in *Scenario 2* of the five-year empirical analysis (01.01.2016 to 01.01.2021), the maximum Sharpe ratio is found in a portfolio with a 0.00% weight of a commodity index (S&P GSCI Commodity Index - Total Return). The same was found in *Scenario 3*, where the maximum Sharpe ratio is found in a portfolio with a 0.00% weight of a commodity index (Rogers International Commodity Index - Total Return RICI), leading to the conclusion that commodities being included in *Scenario 2* and *Scenario 3* did not contribute to a better risk-return ratio or to an overall better portfolio performance at all.

- In the seventeen-year empirircal analysis, when taking a look at the risk-return profiles of all three scenarios, it is evident that the tangency portfolio (i.e. market portfolio) from *Scenario 1* (the one without commodities) has the most superior risk-return profile and the highest Sharpe ratio compared to the other two scenarios including commodities (*Scenario 2* and *Scenario 3*, respectively). This is also supported by the two figures below. Figure 27 and Figure 28, show *Scenario 1* (the scenario without commodities) right next to each of the scenarios including commodities (*Scenario 2* and *Scenario 3*, respectively). A similar conclusion can be drawn when analysing the Sharpe ratios in the five-year empirical analysis (01.01.2016 to 01.01.2021), being that the most superior risk-return profile and the highest Sharpe ratio is found in a portfolio in *Scenario 1* and *3* (a portfolio with the same composition in both scenarios, neither of them including commodity investments at all).

Figure 27

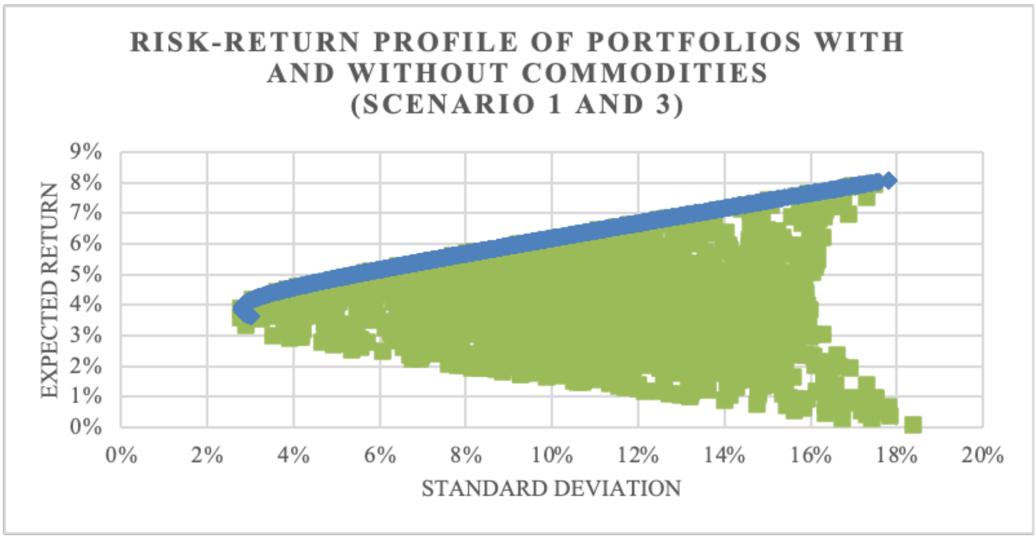
Risk-return profile of portfolios with and without commodities (Scenario 1 and Scenario 2)



Note. Source: Own calculation.

Figure 28

Risk-return profile of portfolios with and without commodities (Scenario 1 and Scenario 3)



Note. Source: Own calculation.

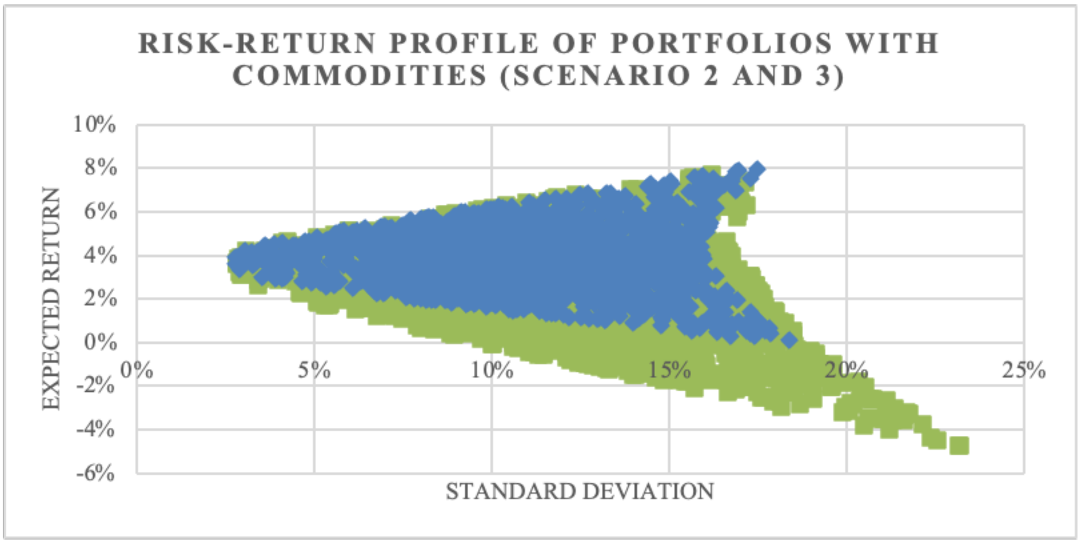
As argued by Fabozzi, Füss and Kaiser (2011), when adding commodities to a portfolio of stocks and bonds, there is a movement of the efficient frontier around the minimum variance portfolio (i.e the treasury bill rate) in a counterclockwise fashion, meaning that increased risk-adjusted returns could be gained by this portfolio. As can be seen in Figure 27 and Figure 28, this is not the case in this empirical analysis, since such movements of the efficient frontier could not be observed.

- When it comes to *Scenario 2*, this is in line with the abovementioned finding, since the maximum Sharpe ratio in *Scenario 2* did not include the investments in the commodity index, the S&P GSCI Commodity Index - Total Return, at all, making it understandable that no counterclockwise movements of the efficient frontier could be observed in Figure 27. Even though, in *Scenario 3*, the highest Sharpe ratio was observed in a portfolio having 0.98% invested in a commodity index, the Rogers International Commodity Index - Total Return RICI, the highest Sharpe ratio of the *Scenario 3* was still lower than the highest Sharpe ratio of *Scenario 1*, i.e. the scenario without commodity investment, making investments without commodities a much superior investment option than without and making it understandable why no counterclockwise movements of the efficient frontier could be observed in Figure 28. A similar conclusion can be drawn when analysing the Sharpe ratios in the five-year empirical analysis (01.01.2016 to 01.01.2021), being that the most superior risk-return profile and the highest Sharpe ratio in all three scenarios were linked to portfolios that did not include commodity investments at all, leading to the conclusion that no better risk-return ratio or overall better portfolio performance is achieved upon including the S&P GSCI Commodity Index - Total Return or the Rogers International Commodity Index - Total Return RICI in a portfolio.

- In the seventeen-year empirical analysis, Scenario 2 (the green area in Figure 29) and Scenario 3 (the blue area) are plotted right next to each other in order to see clearly whether or not there are differences in portfolio optimization depending on which of the two commodity indices is selected (the S&P GSCI Commodity Index - Total Return in Scenario 2 and the Rogers International Commodity Index - Total Return RICI in Scenario 3). Figure 29 shows that Scenario 3 offers marginally better risk-return profiles, since the blue area is slightly over the green one in the upper corner of Figure 29 (in a counterclockwise direction). This conclusion is also supported by the calculations, since the tangency portfolio in Scenario 3 (i.e. market portfolio) has a higher Sharpe ratio than in Scenario 2. However, it should be noted that the tangency portfolio in Scenario 2 (i.e. market portfolio), with the highest Sharpe ratio, is the one with 0.00% invested into the commodity index. Therefore, there are differences in portfolio optimization depending on which commodity index is selected, since the Sharpe ratio from Scenario 3, with the investment in the Rogers International Commodity Index - Total Return RICI, is higher than all other Sharpe ratios from the two thousand randomly chosen protfolios entailing the S&P GSCI Commodity Index - Total Return in Scenario 2. Therefore, based on the findings of this seventeen-year empirical analysis, investing into the Rogers International Commodity Index - Total Return RICI, a highly stable index, with broad and consistent components, is much more favourable than investing in to S&P GSCI Commodity Index - Total Return, the index with a very high share invested in the energy sector, as argued by Beeland Interests (2021) and Boerse. de (2021) respectively. However, since the portfolios with the highest Sharpe ratios in Scenario 2 and Scenario 3 of the five-year empirical analysis (01.01.2016 to 01.01.2021) did not include commodity investments at all, this analysis provides no definite answer on which commodity index is a better choice.

Figure 29

Risk-return profile of portfolios with commodities (Scenario 2 and Scenario 3)



Note. Source: Own calculation.

Therefore, both empirical analyses – the seventeen-year and the five-year one – show a similar trend when it comes to the risk-return profiles, the covariance and correlation matrices (only difference being the positive but low annual rate of return of the Rogers International Commodity Index - Total Return). When it comes to portfolio optimization, a slightly different conclusion can be drawn: including commodities in Scenario 3 did not contribute to a better risk-return ratio or an overall better portfolio performance at all in the five-year empirical analysis, unlike in the seventeen-year one.

In summary, the empirical analysis presented in Chapter 5 does not confirm the hypothesis of this thesis, due to the following:

- When observed on their own merit, commodities were a less attractive investment option in comparison to stocks and bonds, since the annual rates of return of the two commodity indices were very unfavourable (low, even negative in the majority of cases), with very high volatility in both empirical analyses.

- Low correlation of commodity indices was found just in case of the bond index, the FTSE World Government Bond Index - Total Return. High correlations with the stock index, the MSCI World Index - Total Return, were recorded with both commodity indices within both empirical analyses, negating the supposed main advantages of commodities (its very low correlations to equity and combinations of equity and bonds).
- When analysing all three scenarios of the seventeen-year empirical analysis, it is evident that the tangency portfolio (i.e. market portfolio) with the most superior risk-return profile and the highest Sharpe ratio was the one in *Scenario 1* (scenario without commodity investment). Thus, even the portfolio with the highest Sharpe ratio in *Scenario 2* (scenario with commodities) in the same empirical analysis did not include the investments in the commodity index at all. Therefore, aside from the portfolio with the highest Sharpe ratio in *Scenario 3* of the seventeen-year empirical analysis, the one where commodities (Rogers International Commodity Index - Total Return RICI) account for only 0.98% of the portfolio, the portfolio with the superior risk-return profile and the highest Sharpe ratio from the other two scenarios did not include commodity investment at all. In the five-year-empirical analysis, a similar outcome is observed, since the most superior risk-return profile and the highest Sharpe ratio of all three scenarios is found in a portfolio in *Scenario 1* and *3* (a portfolio with the same composition in both scenarios, neither of them including commodity investments at all). Within both, the scenarios *2* and *3* (scenarios including commodities), the portfolio with the superior risk-return profile and the highest Sharpe did not include commodity investments at all, leading to the conclusion that no better risk-return ratio or overall better portfolio performance is observed when including a commodity index (the S&P GSCI Commodity Index - Total Return or the Rogers International Commodity Index - Total Return RICI) in a portfolio at all.

On this evidence, it can not be argued that global investments perform better (have a better risk-to-return ratio) if they include commodities and that including this asset class in the portfolio is likely to enhance its performance, due to positive effects of diversification.

6. Conclusion

The recent years were marked by commodities becoming an increasingly attractive investment opportunity, with an increased variety of investment instruments as well as with poor performance and increased volatility of equities and real estate after the technological bubble in the year 2000 and the economic crises of 2008-2009 (Jensen and Mercer, 2011). Therefore, it is hardly surprising that investing in commodities is becoming increasingly interesting for investors worldwide due to the relatively poor performance of traditional asset classes, followed by an increasing performance of commodity market indices – for instance, that of the Rogers International Commodity Index (RICI), with its astonishing increase in total return of 324.00% in the time period August 1998 until April 2022 (Beeland Interests, 2022). Furthermore, the fact that commodities are traditionally linked to very low correlations to equity and combinations of equity and bonds is contributing to the attractiveness of commodities as an investment option even more (Kayser, Paris and Ross, 2011).

The empirical research of Heidorn and Demidova-Menzel (2011) indicates that gold was an attractive investment opportunity in the time period from the year 2000 onwards, due to its low remaining correlation. The empirical research of Öztek and Öcal (2017) comes to a similar conclusion, suggesting that high gains arise from portfolio diversification between commodity and stock markets as opposed to investing only in the stock market. Moreover, according to the research in question, the portfolio provides better improvements over more quiet periods rather than over more volatile ones and, therefore, the optimal weights of the assets in the portfolio should be tailored in accordance with the market regimes (Öztek and Öcal, 2017). A negative relation between delayed bond returns and gold is highlighted by the empirical research of Narayan, Thuraisamy and Wagner (2017) as well, concluding that bond returns negatively Granger cause oil prices, i.e. high bond returns predict lower oil prices and vice versa. As per the empirical research of Abid, Dhaoui, Goutte and Guesmi (2019), the best hedge for the U.S. equity market is provided by gold, beating bonds and oil in the long run, since these assets (oil, gold and bonds) depend considerably on equities over time, whether positively or negatively, highlighting their hedging benefits further.

On the other hand, the empirical research done by Bredin, Conlon and Potì (2017) argues that differences within the investment horizons exist, as investing in gold in the medium and short term leads to a strong downside risk decrease. However, silver and platinum show strong risk reduction characteristics only in the short term. In the long term, silver and platinum may even lead to a higher downside risk for higher allocations (Bredin, Conlon and Potì, 2017). The empirical research of Belousova and Dorfleitner (2012) argues that risk reduction in a portfolio could be linked to the addition of certain commodities – for instance, adding agricultural, livestock and industrial metal commodities to a portfolio is especially attractive to risk-adverse investors, with a more prominent allocation on the conservative part of the efficient frontier, with portfolios near the global minimum variance portfolio (GMVP). However, in bear markets, weak diversification capabilities are noticed in softs, grains and livestock sectors, whereas industrial metals have almost no diversification capabilities in bull markets. The most valuable addition to a portfolio are commodities coming from the energy and precious metal sectors, since they are beneficial for the portfolio in both bear and bull markets, in terms of both risk and return, making them a good choice for any investor (Belousova and Dorfleitner, 2012). The empirical research of Cotter, Eyiah-Donkor and Potì (2017) shows that adding commodities, currencies or both to a traditional portfolio may lead to diversification benefits, however not applying to the period of the commodity boom, since that was the period when commodities did not have diversification benefits.

Even though Cheung and Miu's (2010) empirical research concludes that commodities provide statistically significant diversification benefits in the long run, it is highlighted that changes in commodity futures' behavior are regime-dependent (low-return commodity futures environment linked to low volatility and vice cersa). In bearish stock markets, diversification benefits of commodities are very low, which leads to the conclusion that the real advantage of commodities lies in them increasing portfolio performance in infrequent outbreaks in the commodity market (Cheung and Miu, 2010). On the other hand, the empirical research of Fethke and Prokopczuk (2018) indicates one further distinction when concluding on the diversification benefits of commodities, outlining that the second and third generation indices are better linked to higher diversification benefits than the first generation.

This thesis' empirical analysis compares prices of four total return indices, all denominated in the same currency (USD), each representing various different markets (commodity, bond and stock market) over a period of seventeen years (02.01.2004 to 01.01.2021). A shorter, five-year empirical analysis (01.01.2016 to 01.01.2021) was additionally conducted in order to challenge the findings of the longer, seventeen-year empirical analysis. Two different commodity indices are used for calculations, in order to compare the effects and differences of different commodity indices on portfolio optimization. The seventeen-year empirical analysis consists of three main parts: the risk and return analysis, the data normality tests and the portfolio analysis, where different portfolio optimization calculations, i.e. three different scenarios (scenarios with and without commodities), are performed and their results analysed. An additional five-year empirical analysis was conducted, with a focus on a risk and return analysis, a correlation and covariance analysis and different portfolio optimization calculations. For all calculated scenarios, the portfolios that should be chosen in the tangency portfolio (i.e. market portfolio) are identified, i.e. the portfolios with the maximum Sharpe ratio, and their portfolio composition analysed (Schulmerich, 2013). When taking a look at the risk-return profiles of all three scenarios within the seventeen-year empirical analysis, it is evident that the tangency portfolio (i.e. market portfolio) from *Scenario 1* (the one without commodities) has the most superior risk-return profile and the highest Sharpe ratio compared to the other two scenarios including commodities (*Scenario 2* and *Scenario 3*, respectively). A similar conclusion was reached after analysing the Sharpe ratios within the five-year empirical analysis (01.01.2016 to 01.01.2021) since the most superior risk-return profile and the highest Sharpe ratio of all three scenarios is found in a portfolio in *Scenario 1* and *3* (a portfolio with the same composition in both scenarios, neither of them including commodity investments at all). Within both scenarios, *2* and *3* (scenarios including commodities), the portfolio with the superior risk-return profile and the highest Sharpe ratio did not include commodity investments at all, leading to the conclusion that no better risk-return ratio or overall better portfolio performance is observed when including a commodity index in a portfolio at all.

The research hypothesis of this thesis, that global investments perform better (have a better risk-to-return ratio) if they include commodities and that including commodities in the portfolio enhances the portfolio performance, due to positive effects of diversification, is put to test in this thesis. The empirical research done in this field of study does not provide a definitive answer – as it depends on the type of commodity, investment horizon, current market situation etc. Some of the research also claims that adding commodities to a portfolio does enhance its performance, as already mentioned. However, the empirical analysis done in this thesis indicates the exact opposite – that the research hypothesis can not be confirmed, i.e. that including commodities in a portfolio does not enhance the portfolio's quality in terms of return and volatility. Therefore, the findings of this thesis are not in line with previous empirical research done in this field and as such offer a good addition to the existing empirical research and the overall argument when it comes to this topic, especially being that it observes portfolio performace over an unconventionally longer time horizon of seventeen years – through both highs and lows of the commodity market.

The main limitation faced in this empirical analysis is using just four indices to represent price movements of three asset groups (stocks, bonds and commodities). This empirical analysis could be further improved by adding even more asset classes and more indices to represent them, since the empirical research of Belousova and Dorfleitner (2012) and Bredin, Conlon and Potì (2017) indicate that differences between various commodities and their influence on portfolio diversification do exist. The addition of more asset classes and more indices representing them would provide an even broader overview of price movements on the market and would make up for a more diversified portfolio. One further limitation of this empirical research is that weekly price movements are used instead of daily price movements. Using daily prices would provide more reliable data for the empirical analysis. In addition, indicators like Jarque-bera could be used to improve the quality of the empirical analysis even further, as suggested in the empirical research of Narayan, Thuraisamy and Wagner (2017).

There is a new trend arising in this field of study, offering increasing evidence that a significant risk premium can be earned through factor-based commodity investing, i.e. an investment strategy based on exposures of commodities' main features, as outlined by Sakkas and Tessaromatis (2020). According to available research, this type of investment strategy outperforms commodity indices like the S&P GSCI Commodity Index - Total Return or a passive commodity index with equal weights, making it a very valuable new investment approach (Sakkas and Tessaromatis, 2020). This thesis could benefit further by the inclusion of calculations from the field of factor-based commodity investing, that could be compared to portfolio optimization by the Markowitzs mean-variance model, in order to compare the two approaches and see how they differ in terms of results (i.e. risk premium).

Since this thesis focuses on the time period from 2004 to 2020, the latest challenges for markets worldwide caused by the COVID-19 pandemic and its effects on commodity investing need to be addressed as well. As the COVID-19 pandemic progressed and the number of people infected increased, the negative influence on stock market returns was observed (Yang and Dang, 2021).

In the beginning of the COVID-19 pandemic, due to the high volatility observed on the oil markets and their declining prices, the question was raised whether the negative correlation between the oil and the stock market will cease to exist, as outlined by Prabheesh, Padhan, Garg (2020). The empirical research by Prabheesh, Padhan, Garg (2020) shows that, during the first months of the COVID-19 pandemic, a positive correlation between the oil and stock markets is noticed, making commodities a less attractive option for portfolio diversification during this period of time.

According to Tarchella, Mzoughi and Bélaïd (2022), the impact of the recent COVID-19 pandemic on global markets has been significant, especially the influence it had on the development on the stocks and commodity markets. As per the empirical research by Tarchella, Mzoughi and Bélaïd (2022), a great impact of the COVID-19 pandemic on returns on the markets, especially when it comes to long-term investments, is noticeable. As a result of the COVID-19 pandemic, a lack of diversification possibilities between stocks (American, European and Chinese) and energy commodities was noticed as well (Tarchella, Mzoughi and Bélaïd, 2022).

Taking into consideration the findings and the direction of the empirical research mentioned above, a thesis like this one could certainly benefit even further by including the years affected by COVID-19 after the year 2020 and therefore significantly lengthening the timeframe of this empirical analysis. This could possibly provide an insight into how this crisis influenced the commodity markets and shed light on the portfolio diversification possibilities arising from it.

Moreover, the most recent developments in the world, such as the raging war in Ukraine, may even further impact the global economy. Changes on energy markets are evident, such as changes in the price and supply of commodity markets of oil, gas, platinum, silver and gold (Alam, Tabash, Billah, Kumar and Anagreh, 2022). These developments are arguably the biggest change on financial markets since the 2008 crisis that impacted the oil and gold market greatly (Alam, Tabash, Billah, Kumar and Anagreh, 2022). The ongoing war in Ukraine forced the European Union to speed up the process of investing in renewable energies, with a goal to increase the percentage of renewable energy in the Euopean Union by the year 2030 even more (Bloomberg, 2022). For example, Germany has a very ambitious goal of relying solely on renewable energy by the year 2035, which will greatly impact the investments in renewable energies like wind and solar panels (Bloomberg, 2022). These changes in investments worldwide will surely impact the demand and supply on commodity markets, leading to further global market changes in this segment.

The timeframe of this empirical analysis includes the financial crisis of 2008 and its impact on the markets and ends with the year 2020, the beginning of the COVID-19 pandemic – however, exactly how much of an impact the pandemic and the post-pandemic years, complicated further by the changes forced upon the markets by the war in Ukraine, will have on the markets in the long-run would be a very interesting topic for additional research in this field of study in a few years' time.

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Appendix 1

Code in RStudio

```
library("readxl")
data <- read_excel("Normality test_MA.xlsx", col_names = TRUE)
head(data)
tail(data)

msci <- data$`MSCI WORLD Total Return Index (USD)`
ftse <- data$`FTSE World Government Bond Index - Total Return (USD)`
snp <- data$`S&P GSCI Commodity Total Return Index (USD)`
rogers <- data$`Rogers International Commodity Total Return Index (USD)`
n <- length(msci)
n <- length(ftse)
n <- length(snp)
n <- length(rogers)

ret_msci <- log(msci[-1]/msci[-n])
ret_ftse <- log(ftse[-1]/ftse[-n])
ret_snp <- log(snp[-1]/snp[-n])
ret_rogers <- log(rogers[-1]/rogers[-n])

M_indices <- cbind(ret_msci, ret_ftse, ret_rogers, ret_snp)
head(M_indices)
```

```
tail(M_indices)

#MSCI World Total Return Index

cor(M_indices)

summary(ret_msci)

mean(ret_msci)

median(ret_msci)

min(ret_msci)

par(mfrow=c(1,2))

hist(ret_msci, breaks=55, freq=FALSE, main="MSCI World Total Return Index (USD) data", xlab="Weekly Returns MSCI World Total Return Index")

lines(density(ret_msci), col="red", lwd=2,lty=2)

msci_bar <- mean(ret_msci)

msci_sd <- sqrt(var(ret_msci))

lines(density(rnorm(20000,msci_bar,msci_sd)), col="forestgreen", lwd=3)


hist(ret_msci, breaks=55, freq=F, ylim=c(0,2), main="MSCI World Total Return Index (USD) data /tail", xlab="Weekly Returns MSCI World Total Return Index")

lines(density(ret_msci), col="red", lwd=2,lty=2)

lines(density(rnorm(20000,msci_bar,msci_sd)), col="forestgreen", lwd=3)

skewness(ret_msci)

kurtosis(ret_msci)

kurtosis(ret_msci, excess=FALSE)


qqnorm(ret_msci)

qqline(ret_msci)
```

```
ks.test(ret_msci, "pnorm")

shapiro.test(ret_msci)

#FTSE Wold Government Bond Total Return Index

summary(ret_ftse)

mean(ret_ftse)

median(ret_ftse)

min(ret_ftse)

cor(M_indices)


par(mfrow=c(1,2))

hist(ret_ftse, breaks=55, freq=FALSE, main="FTSE World Government Bond Total Return Index (USD) data", xlab ="Weekly Returns FTSE World Government Bond Total Return Index")

lines(density(ret_ftse), col="red", lwd=2,lty=2)

ftse_bar <- mean(ret_ftse)

ftse_sd <- sqrt(var(ret_ftse))

lines(density(rnorm(20000,ftse_bar,ftse_sd)), col="forestgreen", lwd=3)

hist(ret_ftse, breaks=55, freq=F, ylim=c(0,10), col="lightgrey", main="FTSE World Government Bond Total Return Index (USD) data/ tail", xlab="Weekly Returns FTSE World Government Bond Total Return Index")

lines(density(ret_ftse), col="red", lwd=2,lty=2)

lines(density(rnorm(20000,ftse_bar,ftse_sd)), col="forestgreen", lwd=3)

skewness(ret_ftse)

kurtosis(ret_ftse)

kurtosis(ret_ftse, excess=FALSE)
```

```
qqnorm(ret_ftse)

qqline(ret_ftse)


ks.test(ret_ftse, "pnorm")

shapiro.test(ret_ftse)

#Rogers International Commodity Total Return Index

summary(ret_rogers)

mean(ret_rogers)

median(ret_rogers)

min(ret_rogers)


par(mfrow=c(1,2))

hist(ret_rogers, breaks=55, freq=FALSE, main="Rogers International Commodity Total Return Index (USD) data", xlab ="Weekly Returns Rogers International Commodity Total Return Index")

lines(density(ret_rogers), col="red", lwd=2,lty=2)

rogers_bar <- mean(ret_rogers)

rogers_sd <- sqrt(var(ret_rogers))

lines(density(rnorm(20000,rogers_bar,rogers_sd)), col="forestgreen", lwd=3)


hist(ret_rogers, breaks=55, freq=F, ylim=c(0,2), xlim=c(-0.2,0.1), col="lightgrey", main="Rogers International Commodity Total Return Index (USD) data/ tail", xlab ="Weekly Returns Rogers International Commodity Total Return Index")

lines(density(ret_rogers), col="red", lwd=2,lty=2)

lines(density(rnorm(20000,rogers_bar,rogers_sd)), col="forestgreen", lwd=3)

skewness(ret_rogers)
```

```
kurtosis(ret_rogers)

kurtosis(ret_rogers, excess=FALSE)


qqnorm(ret_rogers)

qqline(ret_rogers)


ks.test(ret_rogers, "pnorm")

shapiro.test(ret_rogers)

#S&P GSCI Commodity Total Return Index

summary(ret_snp)

mean(ret_snp)

median(ret_snp)

min(ret_snp)


par(mfrow=c(1,2))

hist(ret_snp, breaks=55, freq=FALSE, main="S&P GSCI Commodity Total Return Index (USD) data", xlab ="Weekly Returns S&P GSCI Commodity Total Return Index")

lines(density(ret_snp), col="red", lwd=2,lty=2)

snp_bar <- mean(ret_snp)

snp_sd <- sqrt(var(ret_snp))

lines(density(rnorm(20000,snp_bar,snp_sd)), col="forestgreen", lwd=3)

hist(ret_snp, breaks=55, freq=F, ylim=c(0,2), col="lightgrey", main="S&P GSCI Commodity Total Return Index (USD) data/ tail", xlab ="Weekly Returns S&P GSCI Commodity Total Return Index")

lines(density(ret_snp), col="red", lwd=2,lty=2)

lines(density(rnorm(20000,snp_bar,snp_sd)), col="forestgreen", lwd=3)
```

```
skewness(ret_snp)
kurtosis(ret_snp)
kurtosis(ret_snp, excess=FALSE)

qqnorm(ret_snp)
qqline(ret_snp)

ks.test(ret_snp, "pnorm")
shapiro.test(ret_snp)
```